Optimization of Aperture Efficiency of the Cassegrain Dual Reflector Antenna

Ahcene BOUALLEG[1], Salah REDADAA[1], Nadjim MERABTINE[,2],
Rachid HAMDI[1], Malek BENSLAMA[2]

[1] *LAIG, Département de Génie Electrique, Faculté des Sciences & Sciences de l'Ingénierie*
Université de Guelma, B.P 401, Guelma 24000, Algérie

bouadzdz@yahoo.fr
redasdz@yahoo.fr
rachidahl@yahoo.com

[2] *Laboratory of Electromagnetism and Telecommunications (LET) University of Constantine (Algeria)*

merabtinenadjim@yahoo.fr
ma_benslama}@yahoo.fr

Abstract: The design of cassegrain dual-reflector system is based on ray-optical principles and ignores diffraction effects caused by the finite size of the subreflector. For small reflector sizes, the diffraction effects will reduce the aperture efficiency considerably and must therefore be accounted for in efficiency determinations used for comparison between antenna types and for optimization of mechanical parameters. Such calculations are usually performed employing the uniform geometrical theory of diffraction (UGTD) combined with a physical optics integration over the main reflector. In this paper, an alternative approach based on an asymptotic solution of the proper secondary diffraction integral is presented and applied to conventional Cassegrain antennas.

Keywords Multi-reflector antennas, Cassegrain antenna, Minimum blockage

INTRODUCTION

For many years, the dual-reflector systems have found application in point-to-point communications, satellite communications, and radio astronomy [CLA 1977], [HAN 1961]. The most conventional type of such antenna is the Cassegrain with paraboloidal main reflector and hyperboloidal subreflector [GAL 1964], a design which can provide higher aperture efficiency than prime- focus paraboloids. Higher efficiency still can be obtained by shaping the two reflectors in order to give uniform aperture illumination [KIL 1983], [WIL 1965].

In this paper, an alternative approach based on an asymptotic solution of the proper secondary diffraction integral presented and applied to conventional Cassegrain antennas.

1. Geometry

The geometry of the Cassegrain system is simple and well known, but it is helpful to have at hand those formulas describing the dish contours in terms of the significant antenna parameters. The classical Cassegrain geometry, shown in Fig. 1, employs a parabolic contour for the main dish and a hyperbolic contour for the sub dish. One of the two foci of the hyperbola is the real focal point of the system, and is located at the center of the feed; the other is a virtual focal point, which is located at the focus of the parabola. As a result, all parts of a wave originating at surfaces, travel equal distances to a plane in front of the antenna.

For this antenna, we will in this and the following section obtain the overall aperture efficiency, including both subreflector diffraction and blockage. It should be noted that the method of calculation is very general and can be applied to other antenna types as well.

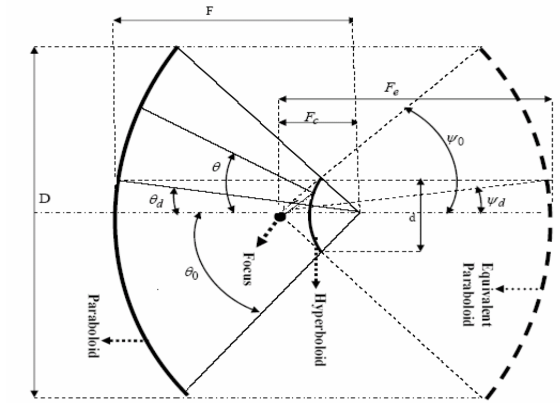

Fig.1 Geometry of cassegrain antenna

Four parameters are needed to define a Cassegrain system. We choose the diameter D and the subtended angle θ_0 of the main reflector, and the diameter d and the subtended angle ψ_0 of the subreflector. We will regard D, θ_0, d, and ψ_0 as independent primary variables, with which to express the aperture efficiency. The following parameters of the antenna are then given in terms of the primary variables by Fig. 1.

$$F = \frac{D}{4}\cot(\theta_0/2) \tag{1}$$

$$F_c = \frac{d}{2}(\cot\theta_0 + \cot\psi_0) \tag{2}$$

$$M = \frac{F_e}{F} = \frac{\tan(\theta_0/2)}{\tan(\psi_0/2)} \tag{3}$$

where F is the focal length of the main reflector, F_c, defines the distance between the primary and secondary focuses, and F_e, is the focal length of the equivalent primary-fed paraboloid [P-S. Kildal] . The relation between an arbitrary feed angle ψ and the corresponding subreflector angle θ is

$$\psi = 2\arctan\left(\frac{1}{M}\tan(\theta/2)\right) \tag{4}$$

The angle ψ_d which determines the center-blocked region of the aperture is given by

$$\begin{aligned}\psi_d &= 2\arctan\left(\frac{1}{M}\frac{d}{4F}\right) \\ &= 2\arctan\left(\frac{d}{D}\tan(\psi_0/2)\right)\end{aligned} \tag{5}$$

We let the feed pattern be specified by its E-plane pattern $A(\psi)$ and its H-plane pattern $C(\psi)$. Then, if the feed is linearly polarized, the complete feed pattern can be written as

$$\vec{E}_f(\psi,\zeta,\rho) = \begin{bmatrix} A(\psi)\sin\zeta\,\vec{a}_\psi \\ +C(\psi)\cos\zeta\,\vec{a}_\zeta \end{bmatrix}\frac{1}{\rho}e^{-jk\rho} \tag{6}$$

where ζ is the azimuth angle of the coordinate system of the feed, and $\vec{a}_\psi$, and $\vec{a}_\zeta$, are unit vectors in the directions of increasing ψ and ζ, respectively.

The first approximation to the aperture efficiency of the Cassegrain is obtained by neglecting subreflector diffraction and blockage. The easiest way to derive it is to use the theory of an equivalent paraboloid reflector. The equivalent paraboloid is defined by [P. W. Hannan]

$$\rho = \frac{F_e}{\cos^2\left(\frac{\psi}{2}\right)}, \qquad \psi \prec \psi_0 \tag{7}$$

Then, from (2), we find

$$\eta_f = \cot^2\left(\frac{\psi_0}{2}\right)\cdot\frac{\left|\int_0^{\psi_0}[A(\psi)+C(\psi)]\tan(\psi/2)d\psi\right|^2}{\int_0^{\pi}\left[|A(\psi)|^2+|C(\psi)|^2\right]\sin\psi d\psi} \tag{8}$$

η_f is often referred to as the feed efficiency. We note that sub- reflector spillover is included in η_f as the power integral in the denominator is taken over all space. An equivalent formula can be obtained by finding the reflected field from the subreflector, and performing the integration over θ instead of ψ. However, it is more convenient to derive the diffraction contribution from (8) because then relations to the feed pattern parameters can be more easily studied.

2. Contributions from subreflector diffraction and blockage

The effects of subreflector diffraction and blockage occur as correction terms to the aperture integral in the numerator of (19). The best way to include them in the aperture efficiency η_a is therefore to define a separate interference efficiency

η_i in such a way that

$$\eta_a = \eta_f \eta_i \tag{9}$$

Where

$$\eta_i = \left|1 + \Delta_{cb} + \Delta_{sb} + \Delta_d\right|^2 \tag{10}$$

The interference terms Δ_{cb}, Δ_{sb} and Δ_d are due to subreflector center blockage, support blockage, and subreflector diffraction, respectively [P-S Kildal].

$$\Delta_{cb} = -C_b \left(\frac{d}{D}\right)^2 \tag{11}$$

Where

$$C_b = \frac{2A(0)\tan^2\left(\frac{\psi_0}{2}\right)}{\int_0^{\psi_0}[A(\psi)+C(\psi)]\tan\left(\frac{\psi}{2}\right)d\psi} \tag{12}$$

$$A(0) = A_0 = \frac{A(\psi_0)+C(\psi_0)}{2A(0)} \tag{13}$$

$$\Delta_d = -(1-j)C_d\sqrt{\frac{\lambda}{d}}\sqrt{1-\frac{d}{D}A0} \tag{14}$$

Where

$$C_d = \frac{1}{2\pi}\frac{\sin\psi_0}{\sqrt{\sin\theta_0}}\frac{2A(0)\tan\left(\frac{\psi_0}{2}\right)}{\int_0^{\psi_0}[A(\psi)+C(\psi)]\tan\left(\frac{\psi}{2}\right)d\psi} \tag{15}$$

is a diffraction parameter which is independent of *d* and *D*. The relation between C_b and C_d is simple,

$$C_d = \frac{1}{\pi}\frac{\cos^2\left(\frac{\psi_2}{2}\right)}{\sqrt{\sin\theta_0}}.C_b \tag{16}$$

In practical antennas we have in most cases that θ_0, close to 90^0 and ψ_0, small, so that

$$C_d \approx \frac{1}{\pi}C_b \tag{17}$$

It should be pointed out that the diffraction loss accounted for by Δ_d in (14) is partly due to main reflector spillover and modified illumination of the main reflector. The main reflector spillover is automatically included in Δ_d as the overall aperture efficiency η_a is normalized to the radiated power from the feed which is the total radiated power (see the denominator of (8)).

If the support blockage (Δ_{sb}) is neglected, the expression for the overall aperture efficiency is given by

$$\eta_a = \eta_f \bullet \eta_i = \left[\frac{2(1-A_0)^2}{-\ln A_0}\right] \bullet \left[\left|1 - C_b\left(\frac{d}{D}\right)^2 - (1-j)C_d\sqrt{\frac{\lambda}{d}}\sqrt{1-\frac{d}{D}A_0}\right|^2\right] \tag{18}$$

Where

$$\frac{d}{D} \approx \sqrt[5]{\frac{1}{(4\pi)^2}\frac{\cos^4\left(\frac{\psi_0}{2}\right)}{\sin\theta_0}A_0^2\frac{\lambda}{D}} \tag{19}$$

Where A_0 is the edge subreflector taper $A_0(dB) = -20\log(A_0)$

This formula is plotted in Fig.2, allowing us to find the optimum d/D, the ratio of subreflector diameter to dish diameter, for any size dish and edge taper.

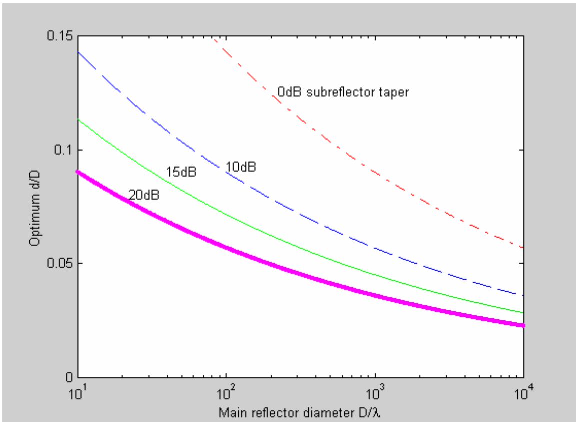

Fig.2 Optimum d/D ratio which maximizes the aperture efficiency of a conventional cassegrain antenna with $\theta_0 = 90^0$ and $\psi_0 = 20^0$

An important result is the interference efficiency η_i as given by (18), i.e. neglecting strut blockage ($\Delta_{sb} \approx 0$). It is presented as a function of *d/D* in Fig. 3 with D/λ as parameter for 10 dB subreflector taper and in Fig. 4 with the taper as parameter for $D = 600\lambda$.

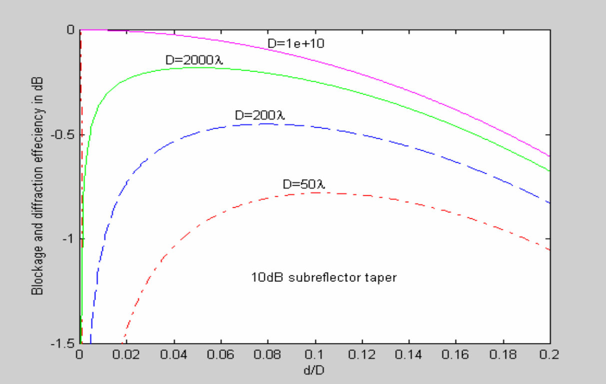

Fig.3 Interference efficiency caused by subreflector blockage and subreflector diffraction for a theoretical feed pattern $\theta_0 = 90^0, \psi_0 = 20^0$, 10dB subreflector taper

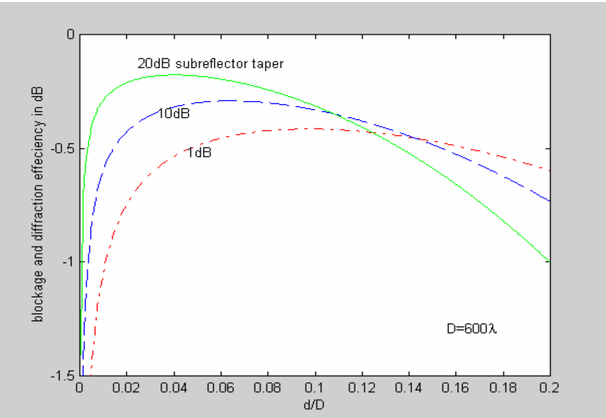

Fig. 4 Interference efficiency caused by subreflector blockage and subreflector diffraction for a theoretical feed pattern $D = 600\lambda$

We find that the maxima correspond to the results in Fig. 2, and that moderate deviations from the optimum d/D cause no significant reduction in efficiency. In Fig. 3 we can study diffraction loss relative to center blockage loss. The curve for $D \to \infty$ includes only center blockage loss which is independent of d/λ so that the difference between this curve and a curve for finite D/λ is due to diffraction loss. We see that the diffraction loss gives a significant correction to the aperture efficiency for $D \prec 200\lambda$.

The maximum efficiency for a given D/λ and for the optimum choice of d/D, can be found by substitution of (19) in (18). The result is

$$\eta_i(\max) = \left[1 - C_b\left(1 + 4\sqrt{1 - \left(\frac{d}{D}\right)}\right)\left(\frac{d}{D}\right)^2\right]^2 \qquad (20)$$

This is plotted in Fig.5 for different subreflector tapers as a function of D/λ. We see that for a 10 dB subreflector taper, the blockage and diffraction loss is larger than *0.6* dB for main reflector diameters less than 100λ

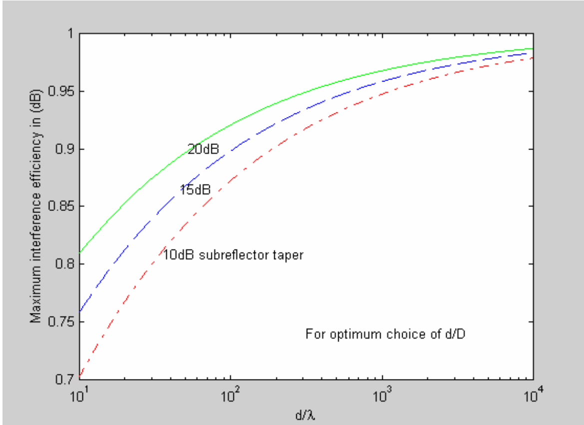

Fig. 5 Maximum possible efficiency due to subreflector blockage and diffraction in a conventional cassegrain for optimum choice of d/D (Figure 2)

3. Conclusion

Optimization of cassegrain dual-reflector antenna it is important to calculate the degradation of the aperture efficiency. This calculation is considerably simplified by the general asymptotic theory which has been presented in this paper. The theory gives a correction to the geometric optics aperture efficiency integral, which is expressed as a well-behaved line integral taken along the main reflector rim.

In this paper the theory has been used to obtain the aperture efficiency of a conventional Cassegrain, including the effects of subreflector diffraction and center blockage. These formulas have been used to find an optimum *d/D* ratio, which maximizes the total aperture efficiency.

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