Quality Improvement of Unbalanced Three-phase Voltages Rectification

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Abstract: In this paper we treat the case where unbalance is on the level of the power supply, and present a method to compensate this unbalance of the network. This method is based on considering a general three phase voltages of unequal amplitude and arbitrary phase. This voltage system is then composed in two sequences: positive and negative sequence. The zero sequence is assured to be zero. For each sequence, a modulation switching function is determined. The voltage in the dc side is then expressed as a function of two sequences voltage, and the related switching. In the above DC voltage expression, there are terms that generate harmonics, we make choice that the sum of harmonic terms equals zero. A global switching function is then determined.

Key words: PMW rectifier, unbalance voltage, unit power factor (UPF).

INTRODUCTION

The electrical network subjected to an environment some time severe, is the seat of incidents which cause disturbances of the power supply due in general to asymmetries of impedances of the lines of the network, with the various effects of short-circuit of origins affecting the network, with the unbalance of the source, the strong single-phase loads or a bad distribution of the loads on the electrical network. When such a defect occurs, there is birth of non symmetry of important voltage which can endanger the safety of the people and deteriorate the existing installations in the electrical network, if it is not quickly eliminated [CHE 06].

This paper proposes a new control scheme for the ac/dc three-level PWM rectifier under generalized unbalanced operating conditions. The quantification of this unbalance is based on the voltages decomposition according to the symmetrical component of Fortescue [GRA 03]. This method permits to separate the three-phase unbalanced voltages on three balanced independent systems known as direct (positive sequence), opposite (negative sequence) and homopolar.

Simulation was conducted under two different cases of unbalanced operating conditions. The first case is 39% unbalance in one phase input voltage. The second unbalance case is a double sinusoidal input. This second unbalance case is one example of the extreme unbalanced operating conditions. This particular operating condition is also commonly found in residential areas where both 110Vac and 220Vac input voltages are available [SUH 02].

A detailed simulation program is prepared by using Matlab/Simulink. Simulation and experimental results confirm the proposed control method.
1. Rectifier with commutation forced with source of unbalance voltage: compensation of unbalance

Considering a three-phase input voltages (unequal amplitudes and arbitrary phases) without a zero sequence as the orthogonal sum of positive and negative sequence, therefore the input can write the entry like vector of three elements:

\[ v_i = \begin{bmatrix} v_{ag} \\ v_{bg} \\ v_{cg} \end{bmatrix} \quad (1) \]

\[ v_{ag} = V_{ag} \cos(\alpha + \phi_a) \]
\[ v_{bg} = V_{bg} \cos(\alpha + \phi_b) \]
\[ v_{cg} = V_{cg} \cos(\alpha + \phi_c) \quad (2) \]

According to the equation of Euler, each voltage of phase can be represented by the sum of two complex sizes, for example, for phase 1:

\[ v_{ag} = \frac{1}{2} \left( V_{ag} e^{j(\alpha + \phi_a)} + V_{ag} e^{-j(\alpha + \phi_a)} \right) \quad (3) \]

\[ v_{ag} = \frac{1}{2} V_{ag} e^{j\phi_a} + \frac{1}{2} V_{ag} e^{-j\phi_a} = j\omega \quad (4) \]

\[ \bar{V}_{ag} = V_{ag} e^{-j\phi_a} \quad (6) \]
\[ \bar{V}_{ag} = V_{ag} e^{j\phi_a} \quad (7) \]

The same formalism is applied to other phases. The space phasor of the three-phase system can be expressed as:

\[ \bar{v}_{1} = \frac{2}{3} \left( \bar{V}_{ag} + j\omega \bar{V}_{bg} + \bar{V}_{cg} \right) \quad (8) \]

\[ \bar{v}_{l} = \frac{1}{3} \left( \bar{V}_{ag} + j\omega \bar{V}_{bg} + \bar{V}_{cg} \right) e^{-j\phi_{l}} \quad (9) \]

We notice that the tension \( \bar{v}_{l} \) is composed of a positive sequence and a negative sequence of voltage. The positive sequence is defined as:

\[ v_{ap} = \frac{1}{3} \text{Re} \left[ \left( \bar{V}_{ag} + j\omega \bar{V}_{bg} + \bar{V}_{cg} \right) e^{j\omega} \right] \quad (10) \]

Therefore, the three voltages of the positive sequence are written:

\[ v_{ap} = \frac{1}{3} \text{Re} \left[ V_{ag} e^{j\phi_a} + V_{bg} e^{j\phi_b} + V_{cg} e^{j\phi_c} \right] \quad (11) \]

And the output voltage continuous side can be described with the following equation:

\[ V_{dc} = \frac{1}{2} \left( v_{ma} + v_{mb} + v_{mc} \right) \quad (15) \]

Where \( m_a, m_b \) et \( m_c \) are the switching functions relative to the three arms of the PWM rectifier, which can take the value (1) or (-1), equation (16) can be written in vectorial form:

\[ 2V_{dc} = m_{ap} V_{ip} + m_{bp} V_{ip} + m_{cp} V_{ip} \quad (16) \]

The first two terms produce the constant voltage \( V_{dc} \), while the two other terms produce harmonic 2 at the frequency of the line. [GRA03]

If the tensions of the positive sequence are written:

\[ v_{ap} = V_{ap} \cos \left( \alpha + \phi_p \right) \quad (17) \]

The corresponding switching functions are then...
\[
m_{ap} = \cos\left(\alpha + \phi + \phi_{mp}\right)
\]
\[
m_{bp} = \cos\left(\alpha + \phi + \frac{2\pi}{3} + \phi_{mp}\right)
\]
\[
m_{cp} = \cos\left(\alpha + \phi + \frac{4\pi}{3} + \phi_{mp}\right)
\]

What produces:
\[
m_T = \frac{1}{V_{mp}} \frac{v_T}{ip} \left(\alpha + \phi_{mp}\right)
\]

After calculation, we find:
\[
m_{ip} = \frac{3}{2} V_{mp} \cos\left(\phi_{mp}\right)
\]

The same manner for the negative sequence, if we take as example:
\[
v_{an} = V_{mn} \cos\left(\alpha + \phi + \phi_{mn}\right)
\]
\[
v_{bn} = V_{mn} \cos\left(\alpha + \phi + \frac{4\pi}{3} + \phi_{mn}\right)
\]
\[
v_{cn} = V_{mn} \cos\left(\alpha + \phi + \frac{2\pi}{3} + \phi_{mn}\right)
\]

And
\[
m_{an} = -\cos\left(\alpha + \phi + \phi_{mn}\right)
\]
\[
m_{bn} = -\cos\left(\alpha + \phi + \frac{4\pi}{3} + \phi_{mn}\right)
\]
\[
m_{cn} = -\cos\left(\alpha + \phi + \frac{2\pi}{3} + \phi_{mn}\right)
\]

We find:
\[
m_{in} = \frac{1}{V_{mp}} \frac{v_T}{in} \left(\alpha + \phi_{mn}\right)
\]
\[
m_{in} = -\frac{3}{2} \frac{V_{mn}^2}{V_{mp}} \cos\left(\phi_{mn}\right)
\]

If these quantities are inserted into formula (19), the last two terms it yields:
\[
m_{ip} + m_{ip} = \frac{v_T}{ip} \frac{v_T}{in} - \frac{V_{mp}}{V_{mn}} \frac{v_T}{ip}
\]

If we choose:
\[
\phi_{mp} = \phi_{mn}
\]

Then
\[
\frac{v_T}{ip} \frac{v_T}{in} = \frac{v_T}{ip}
\]

Thus the two undesirable terms are cancelled between them. And the formula (19) becomes:
\[
2V_{dc} = m_{ip} v_{ip} + m_{ip} v_{in}
\]
\[
2V_{dc} = \frac{3}{2} \left\{ \frac{v_{2m}}{v_{mp}} \cos\left(\phi_{mp}\right) \right\}
\]

Therefore, the function of modulation which permits to produce a continuous and constant voltage for any condition causing an unbalance is:
\[
m_i = \frac{v_{ip} - v_{in}}{V_{mp}} \Leftrightarrow
\]

We note that only the amplitude of the positive sequence \(V_{mp}\) appears in the expression (31), which can be measured by filtering.

The expression (31) is used to develop the diagram of the valve block of the Fig. 2.

Non-symmetry in voltage is characterized by its degree which is defined by using the method of the components of Fortescue by the opposite of the ratio of the amplitudes of the inverse and direct voltage [CHE 06], therefore the factor of non-symmetry or unbalance is defined by the following relationship:
\[
U = \frac{v_{mn}}{v_{mp}}
\]

Then equation (32) becomes:
\[
2V_{dc} = \frac{3}{2} v_{mp} \left(1 - U^2\right) \cos\left(\phi_{mp}\right)
\]

Therefore, the expression of the maximum value of the voltage \(V_{dc}\) is:
\[
2V_{dc} = \frac{3}{2} v_{mp} \left(1 - U^2\right)
\]

Equation (34) shows the relationship between the continuous voltage and the factor of unbalance, as long as this last increases, which induces reduction on the level of dc voltage and reciprocally.

The Fig. 2 shows the evolution of the maximum value of the voltage \(V_{dc}\) according to the factor of unbalance.
2. Simulation Results and discussions

To prove the feasibility and the performance of the proposed control scheme under various generalized unbalance operating conditions, simulation was made for two different unbalanced operating conditions.

Case 1: 39% of unbalance in one phase input voltage

This kind of unbalanced input condition is quite common in a weak ac system [SUH 02]. The parameters of our system for this condition are summarized in Table 1.

Table 1. Parameters used in simulation for case 1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
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<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_x, L_y, L_z$</td>
<td>8 mH</td>
<td>$R_x, R_y, R_z$</td>
<td>0.3 Ω</td>
</tr>
<tr>
<td>$v_a$</td>
<td>220√2 sin(αV)</td>
<td>$V_{ac}$</td>
<td>700 V</td>
</tr>
<tr>
<td>$v_b$</td>
<td>220√2 sin(α - 120°) V</td>
<td>$C_d$</td>
<td>5000 μF</td>
</tr>
<tr>
<td>$v_c$</td>
<td>134√2 sin(α + 120°) V</td>
<td>$R_c$</td>
<td>10 Ω</td>
</tr>
</tbody>
</table>

Fig. 3a illustrates the three phase voltages, where the voltage has an unbalance of 39% in the amplitude. Fig. 3b shows the phase (a) input current, and the phase (a) input voltage, finally, Fig. 3c illustrates the converter output dc voltage.
According to Fig. 3b, we see that the current is practically sinusoidal and in phase with the voltage of the phase (a). Fig. 3c shows that the dc voltage follows its reference.

**Case II: A double sinusoidal input**

Although this type of unbalanced operating condition is not as common as that of case 1, a double sinusoidal input is considered to be one of the most extreme unbalanced operating conditions. Consequently, it is a suitable example to examine the performance capability of a proposed method. The system parameters of this condition are recapitulated in Table 2.

<table>
<thead>
<tr>
<th>Paramètres</th>
<th>Valeur</th>
<th>Paramètres</th>
<th>Valeur</th>
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<tbody>
<tr>
<td>$I_{p1}$, $I_{p2}$</td>
<td>4m/1</td>
<td>$R_s$, $R_a$</td>
<td>0.6Ω</td>
</tr>
<tr>
<td>$I_1$</td>
<td>8m/1</td>
<td>$V_{dc}$</td>
<td>70V</td>
</tr>
<tr>
<td>$v_{a1}$</td>
<td>$220\sqrt{2}\sin(\omega t)$</td>
<td>$C_{dc}$</td>
<td>5000µF</td>
</tr>
<tr>
<td>$v_{b2}$</td>
<td>$220\sqrt{2}\sin(\omega t - 120^\circ)$</td>
<td>$R_a$</td>
<td>1Ω</td>
</tr>
<tr>
<td>$v_{a2}$</td>
<td>0V</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 2. Parameters used in simulation for case 2**

Fig. 4a presents the three voltages, Fig. 4b illustrates the voltage and the current of the phase (a), we see that the current is practically sinusoidal and in phase with the voltage, finally the Fig. 4c shows that the voltage follows its reference but with the presence of the ripple.

**Figure 4. Results of simulation in the case of 39% of unbalance on the level of the amplitude : (a) three phase voltages; (b) voltage of the phase (a); (c) current of the phase (a); (d) DC voltage.**

**Figure 5. Results of simulation in the case of cut on the level of the third phase: (a) three phase voltages; (b) voltage of the phase (a); (c) current of the phase (a); (d) DC voltage.**
3. Conclusion

This paper proposes a new method for three-level PWM rectifier operating under generalized unbalanced operating conditions. An extreme unbalanced operating condition of input voltage as one of two example conditions for the simulation in this paper. The dc output and nearly unity power factor in ac side of the rectifier show the effectiveness of the method.

REFERENCES

