

# Dispersion Characteristics of Microstrip Lines

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**Abstract:** This paper presents an efficient hybrid method namely spectral domain approach (S.D.A) for the analysis of dispersion characteristics of microstrip lines. The analysis is based on the Green's functions/moment method, where the electric-field integral equation, representing the boundary conditions that the total field must vanish on the perfect electric conductors of the structure, is discretized by the Galerkin procedure. The unknown coefficients are evaluated by solving the resulting matrix equation.

**Key words:** Microwave circuits, Microstrip lines, Integral equations, Spectral domain method.

## INTRODUCTION

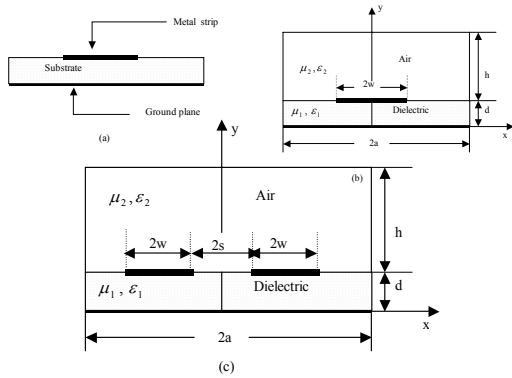
Planar transmission structures are widely used in microwave, millimeter-wave circuits and high speed digital circuits. These are striplines, microstrips and coplanar waveguides [COL 1987], [GUP 1979], [TOU 2005]. Spurious radiation may occur at discontinuities of the microstrip lines. Moreover, it is a common situation that they can interfere with other devices or lines placed in the same environment. To this purpose, the electromagnetic investigation is particularly important in the time domain, where we have transient phenomena in response to an impulse excitation. In this case the analysis becomes more complicated, in particular when a lot of devices is present in the same environment. The presence of these impulsive fields provides a great amount of electromagnetic disturbance against which the microstrip should be immune. With that the analysis of the microstrip susceptibility becomes an important requirement for its design. In this contribution we present an analysis of single and coupled microstrip lines.

The application of the Fourier transform can solve many of the difficulties with the space domain integral equation method, and leads to the so-called Spectral Domain Method (SDM). This technique was first developed for shielded microstrip lines by Itoh and Mittra [ITO 1974], and has since been extended to a large number of transmission line structures [TSA 2003]. The SDM is essentially a procedure for computing the Green's function for the problem by Fourier decomposition, reducing the problem to that of solving algebraic equations. Provided that the basis set for the Moment Method solution can be transformed analytically, it reduces the computation burden to the evaluation of integrals or series in one dimension. The key requirements of the Spectral Domain Method are the existence of planar dielectric regions which are uniform in lateral extent, and infinitesimally thin perfect conductors which lie along a dielectric interface.

## 1. Microstrip lines

A microstrip line consists of a substrate, a ground plane and a metal strip etched on the top of the

substrate as shown in Fig.1.a. Conductors are usually gold plated and the substrate material, depending on the application requirements, can be a dielectric, a ferrite, a semiconductor or something else. As part of a circuit, the microstrip line together with other circuit components are enclosed within a metal box as shown in Fig.1.b. The function of the box is to provide the electromagnetic shielding as well as to protect the circuit against shocks and vibrations. In studying microstrip lines, the size and the shape of the box are important factors.



**Fig.1.** Cross-section of (a) an open microstrip line (b) a shielded microstrip line (c) coupled strip line.

In the analysis, we consider a microstrip line with the cross-section shown in Fig.1.b. The dispersion expression can be found upon the successful solution of Maxwell's equations, subject to boundary conditions for the structure of concern [XUN 1990]. Since the microstrip line can only support hybrid modes (TE and TM modes simultaneously) [MIT 1974], two scalar potential functions  $\phi_i$  and  $\psi_i$  are required in order to obtain the full-wave solution, where the subscript  $i = 1, 2$  indicate the two dielectric media. These functions are solutions of the wave equations:

$$\nabla_t^2 u_i + (k_i^2 - \beta^2)u_i = 0 \quad (1)$$

where

$$\nabla_t = \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j}$$

$$k_i^2 = \omega^2 \mu_i \epsilon_i$$

$\omega$  is the angular frequency and  $\beta$  denotes the phase constant of the wave in the z-direction.

The Fourier series theory allows the above functions to be expanded in terms of trigonometric

functions of  $x$  as follows [CHE 1992], [HOO 1999], [TAM 2007], and [YAN 1990]:

$$\phi_i = \sum_{n=-\infty}^{+\infty} \tilde{\phi}_i(\alpha_n, y) e^{-j\alpha_n x} \quad (2)$$

$$\psi_i = \sum_{n=-\infty}^{+\infty} \tilde{\psi}_i(\alpha_n, y) e^{-j\alpha_n x}$$

where  $\alpha_n$  is the Fourier parameter which is found by examining the field behaviour along the  $x$ -direction.

The field component  $E_{z,i}$  in the  $i^{\text{th}}$  dielectric can be expressed as

$$E_{z,i} = (k_i^2 - \beta^2) \left[ \sum_{n=0}^{+\infty} \tilde{\phi}_i(\alpha_n, y) e^{-j\alpha_n x} + \sum_{n=0}^{+\infty} \tilde{\psi}_i(\alpha_n, y) e^{-j\alpha_n x} \right] e^{-j\beta z} \quad (3)$$

since  $E_{z,i}$  is an even function of  $x$ ,  $\phi_i(\alpha_n, y)$  will be an even function of  $\alpha_n$ . Hence, the alternative form of Eq.(3) is

$$E_{z,i} = 2(k_i^2 - \beta^2) \sum_{n=0}^{+\infty} \tilde{\phi}_i(\alpha_n, y) \cos(\alpha_n x) e^{-j\beta z} \quad (4)$$

We note that at the dielectric interface, the longitudinal section electric (LSE) and longitudinal section magnetic (LSM) modes are coupled to satisfy the boundary conditions. This gives us the fundamental equation of the spectral method form of matrix connecting the tangential electric field components ( $E_x, E_z$ ) in the Fourier domain and the surface current densities ( $J_x, J_z$ ) in the tangential plane. Thus, we obtain

$$\begin{aligned} \tilde{E}_z &= G_{11}(\alpha_n, y) \tilde{J}_x + G_{12}(\alpha_n, y) \tilde{J}_z \\ \tilde{E}_x &= G_{21}(\alpha_n, y) \tilde{J}_x + G_{22}(\alpha_n, y) \tilde{J}_z \end{aligned} \quad (5)$$

$J_x(x), J_z(x)$  and  $\beta$  are the unknown of the system.

The matrix  $[G(\alpha_n, y)]$  is known as the dyadic Green's function, its elements  $G_{ij}$  are a function of the geometrical and electric parameters of the structure as well as the frequency and the constant of propagation. to calculate these functions, we used the method known as " Immitance approach" introduced into the

literature by Itoh [ITO 1980], [PRE 2003] which has the advantage of determining the Green's functions of the multi-layer and multi-conductor structures without passing by the calculation of the potential coefficients. The resolution of the system of equations uses the method of the moments (Galerkin method), with a suitable choice of the basis functions for  $J$  (or  $E$ ) and led to the equation of the sought dispersion from which one can determine the characteristics of propagation of the studied lines.

## 2. Galerkin technique

Within the framework of the technique of Galerkin, one starts first of all, by developing the components of the current transformed on the strips like linear combination of known basis functions in the following form [BHA 1995]:

$$\begin{aligned} \tilde{J}_x(x) &= \sum_{r=1}^R a_r \tilde{J}_{x,r}(x) \\ \tilde{J}_z(x) &= \sum_{m=1}^M b_m \tilde{J}_{z,m}(x) \end{aligned} \quad (6)$$

The basis functions are selected so that their inverse Fourier transforms are non-zero over the strip.

On the perfect conductors, the tangential electric field is equal to zero and it is the same for the current density on the dielectric. Therefore the current and the electric field in the plane of metallization are defined in two complementary spaces.

This property makes it possible to write:

$$\int_{-\infty}^{+\infty} \tilde{J}_x \tilde{E}_x dy = 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} \tilde{J}_z \tilde{E}_z dy = 0 \quad (7)$$

By using this complementarity of the boundary conditions as well as the Parseval's identity, we obtain finally the following system:

$$\begin{aligned} \sum_{r=1}^R C_{r,m}^{1,1}(\beta) a_r + \sum_{m=1}^M C_{m,m}^{1,2}(\beta) b_m &= 0, m' = 1, 2, \dots, M \\ \sum_{r=1}^R C_{r,r'}^{2,1}(\beta) a_r + \sum_{m=1}^M C_{m,r'}^{2,2}(\beta) b_m &= 0, r' = 1, 2, \dots, M \end{aligned} \quad (8)$$

where

$$\begin{aligned} C_{r,m}^{1,1} &= \sum_n G_{11}(\alpha_n, \beta) \tilde{J}_{z,m}^*(x) \tilde{J}_{x,r}(x) \\ C_{m,m}^{1,2} &= \sum_n G_{12}(\alpha_n, \beta) \tilde{J}_{z,m}^*(x) \tilde{J}_{z,m}(x) \\ C_{r,r'}^{2,1} &= \sum_n G_{21}(\alpha_n, \beta) \tilde{J}_{x,r}^*(x) \tilde{J}_{x,r'}(x) \\ C_{m,r'}^{2,2} &= \sum_n G_{22}(\alpha_n, \beta) \tilde{J}_{x,r}^*(x) \tilde{J}_{z,m}(x) \end{aligned} \quad (9)$$

We obtain an algebraic system of  $(R + M)$  homogeneous linear equations according to  $(R + M)$  unknown coefficients  $a_r$  and  $b_m$ . The nontrivial solutions of the system of homogeneous equations Eq.(8) provide at a given frequency  $f$ , the constants of propagation of the modes guided by the structure. The nontrivial solutions are obtained by cancelling the determinant of the matrix.

$$\det[C(\beta)] = 0 \quad (10)$$

The equation (10) represents the characteristic equation of the system. Its resolution makes it possible to calculate  $\beta$  at a given frequency and thus the constant of propagation.

## 3. Choice of basis functions

To be effective, this choice must fill a certain number of criteria:

- The selected basis functions for the current must be non-zero over the strip only.
- This choice must take account of the singular behaviour of the electromagnetic field in the vicinity of the edges.
- The expansion of the current density must be done in a complete base.

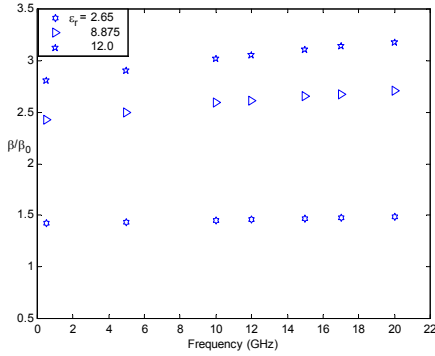
In order to ensure an optimal convergence, we used the following functions which were tested from the viewpoint precision and convergence. For the strip lines, we have:

$$\begin{aligned} J_{x,r}(x) &= \frac{\sin(r\pi x/w)}{\sqrt{1-(x/w)^2}} \\ J_{z,m}(x) &= \frac{\cos[(m-1)\pi x/w]}{\sqrt{1-(x/w)^2}} \end{aligned} \quad (11)$$

## 4. Results & discussions

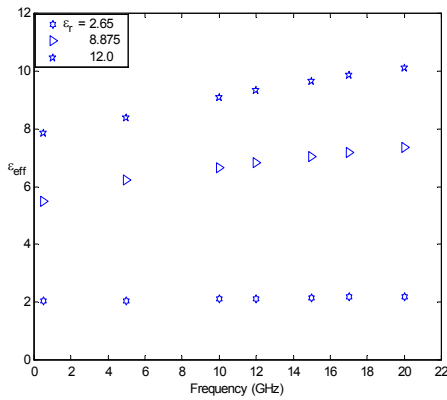
### 4.1 Single Strip line

Fig.2 presents the dispersion characteristics of the fundamental mode for various values of relative permittivity.  $N$  Indicates the number of Fourier terms and  $IN$  the total number of basis functions ( $R + M$ ).



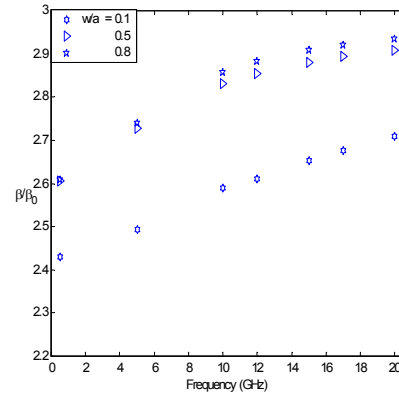
**Fig.2.** Dispersion characteristics of the fundamental mode for various values of relative permittivity with  $N=180$ ,  $IN=8(a=6.35 \text{ mm}, w=0.635 \text{ mm}, d=1.27 \text{ mm}, h=11.43 \text{ mm})$ .

In Fig.3, we show the variation of the effective dielectric constant for the same parameters.

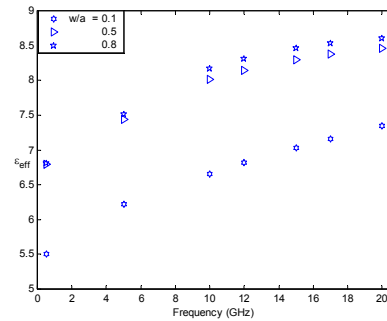


**Fig. 3.** Effective dielectric constant for various values of relative permittivity.

Figs.4 and 5 show dispersion characteristics and the effective dielectric constant for various values of  $w/a$  with a relative permittivity  $\epsilon_r = 8.875$ .



**Fig.4.** Dispersion characteristics of the fundamental mode for various values of  $w/a$ .

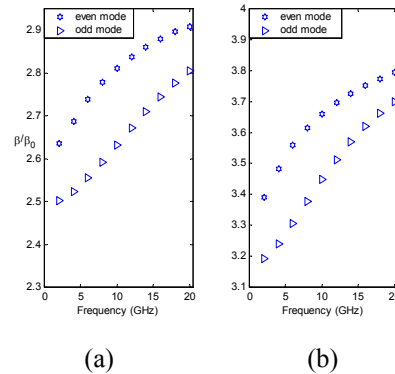


**Fig.5.** Effective dielectric constant for various values of  $w/a$ .

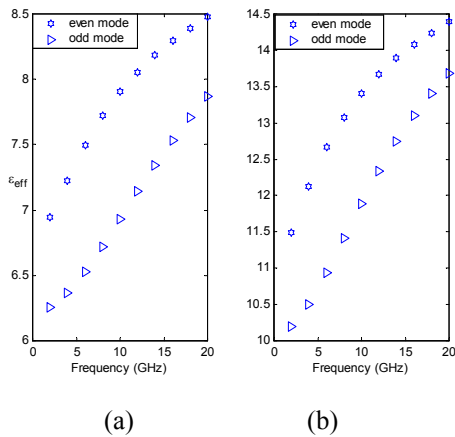
We have used the Gauss elimination technique to compute the values of the determinant of the final matrix. We have found that spurious solutions appear for high frequencies and high permittivity of the substrate.

### 4.2 Coupled Strip lines

We have plotted the dispersion characteristics and the effective dielectric constant for various values of relative permittivity for the two modes (even and odd).

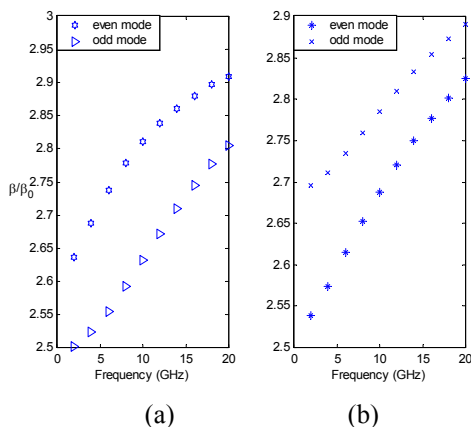


**Fig.6.** Dispersion characteristics of the even and odd modes for various values of relative permittivity (a)  $\epsilon_r = 9.6$  (b)  $\epsilon_r = 16$ .

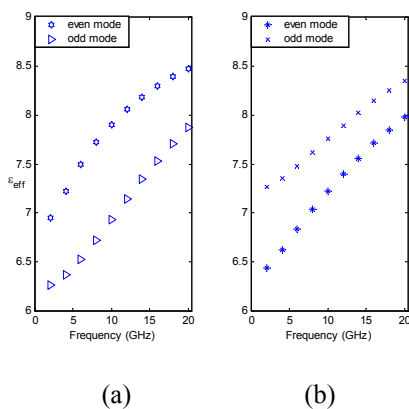


**Fig.7.** Effective dielectric constant of the even and odd modes for various values of relative permittivity (a)  $\epsilon_r = 9.6$  (b)  $\epsilon_r = 16$ .

We have examined the influence of the structure parameters on the dispersion characteristics for a given relative permittivity.



**Fig.8.** Dispersion characteristics of the even and odd modes (a)  $2s/d = 1$ , (b)  $2s/d = 4$  ( $\epsilon_r = 9.6$ ).



**Fig.9.** Effective dielectric constant of the even and odd modes (a)  $2s/d = 1$ , (b)  $2s/d = 4$  ( $\epsilon_r = 9.6$ ).

In the above curves, we note that the variation is less important at the very high frequencies between the two modes.

### 5. Conclusion

In this paper, we have presented an analysis of dispersion characteristics of the microstrip lines by the method of approach in the spectral domain. Under special circumstances, the differential equations governing the behaviour of the structure can be transformed into this domain, where the transformed ones have an easy solution. We have introduced the Green's functions which present simple forms in the Fourier domain. The approach was found to be very efficient and easy to use, and is suitable for a variety of basis functions including mixed-type basis function sets. The precision can be systematically improved by increasing the size of the matrix associated with the system with linear equations.

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