An Analysis of Topographic InSAR Algorithms

S. REDADDA*, A. BOUALLEG*, R. HAMDI*,
N. MERABTINE** & M. BENSLAMA**

*LAIG, Département de Génie Electrique, Faculté des Sciences et Sciences de l’Ingénierie
Université de Guelma, B.P 401, Guelma 24000, Algérie
redasdz@yahoo.fr
rachidahl@yahoo.com
bouadzdz@yahoo.fr

**LET, Département d’Electronique, Faculté des Sciences de l’Ingénieur
Université Mentouri Constantine (UMC), Constantine 25000, Algérie
merabtinenadjim@yahoo.fr
ma_benslama@yahoo.fr

Abstract: Interferometric synthetic aperture radar (InSAR) is a method which provides a means of estimating global topography with high spatial resolution and height accuracy. The general formulation, giving relationship between the interferogram phase and the target height, is based on the interferometric SAR geometry and on a better expansion of the path length difference between the sensor and the target. To derive an estimate of the surface height, this technique uses the relative phase difference between two coherent SAR images obtained by two receivers separated by a cross-track baseline. Three measurements are required to reconstruct the three-dimensional coordinate of a point in an InSAR image: the range, azimuth, and elevation.

In this paper, we focus on the study of topography reconstruction based on the orthogonal decomposition of the look vector of radar. We present an analytical analysis of Madsen geolocation algorithm in which the coordinates of the geolocated point are given in a geocentric Cartesian reference system.

Key words: Algorithms, InSAR, orthogonal decomposition, topography reconstruction.

INTRODUCTION

Interferometric radar has been proposed and successfully demonstrated as a topographic mapping technique by Gabriel and Goldstein [GAB 1988], Graham [GRA 1974], and Zebker and Goldstein [ZEB 1986]. A radar interferometer is formed by relating the signals from two spatially separated antennas; the separation of the two antennas is called the baseline. The spatial extent of the baseline is one of the major performance drivers in an interferometric radar system: if the baseline is too short the sensitivity to signal phase differences will be so small to be undetectable, while if the baseline is too long additional noise due to spatial decorrelation corrupts the signal.

The elevation of the terrain can be derived from the phase difference (interferometric phase) between corresponding pixels in the two SAR images by means of simple geometric relationships [ROD 1992]. From the complex interferogram, the interferometric phase is only known modulo $\frac{\pi}{2}$. In order to be able to relate the interferometric phase to the topographic height, the correct multiple $2\pi$ has to be added. This is done in the phase-unwrapping step after removing the flat-earth expected phase from the interferogram [GHI 1994], [GOL 1988]. The interferometric coherence map, generated by calculating the normalized interimages correlation, characterizes the uncertainty with which the height can be measured. However, successful phase-to-height conversion requires relative orbital parameters accuracy [MOC 1994] and a fulfilment of the basic requirements for repeat-pass interferometric system. These requirements consist of having, first, stable atmospheric conditions and terrain backscatter and, second, the same interferometric SAR geometry during
both SAR acquisitions. Phase-to-height conversion is a very important step in the digital elevation model (DEM) generation procedure. Therefore, the relationship between phase and height should be calculated accurately.

Two distinct implementation approaches have been developed for topographic radar interferometers; they differ in how the interferometric baseline is formed. In the first case the baseline is formed by two physical antennas which illuminate a given area on the ground simultaneously; this is the usual approach for aircraft implementations where the physical mounting structures may be spaced for sufficient baseline, Fig.1(a). This approach is used by [ZEB 1986] for the NASA CV-990 radar, and is also used in the TOPSAR topographic mapping radar mounted on the NASA DC-8 aircraft [ZEB 1992]. The second type of implementation utilizes a single satellite antenna in a nearly-exact repeating orbit. Then it forms an interferometer baseline by relating radar signals on the repeat passes over the same site. Topographic maps using this technique have been demonstrated by Goldstein et al [ZEB 1986] and Gabriel et al [GAB 1988]-[GAB 1989], Fig.1(b).

Here \( H \) is the aircraft altitude, \( b \) is the baseline length, \( \theta \) is the look angle, \( r \) is the slant range, \( \delta r \) is the slant range difference, \( z(x) \) is the topographic height, \( \alpha \) is the baseline orientation angle with respect to the horizontal plane, and respectively its horizontal and vertical components are \( b_x \) and \( b_z \). \( P(X,0) \) and \( Q(X + x,z) \) and are two scatter targets respectively on and above the ground reference.

Although, interferometric topographic mapping is possible using each approach, a significant advantage accrues from using the single-pass aircraft implementation over the repeat-orbit spacecraft method. Specifically in the first approach, we avoid problems associated with temporal decorrelation of the surface, that is, the result that changes on the wavelength scale of the surface lead to additional decorrelation noise in the interferogram. If the change on the surface is large, as could happen if, precipitation occurred, the phases of the received signals may be wholly unrelated. On the other hand, spaceborne platforms provide views of inaccessible regions of the Earth.

The Across-track interferometric technique exploits the phase differences of at least two complex-valued SAR images acquired from slightly different positions and/or at different times to extract height information of topography. It can measure the slant range difference at the order of sub-wavelength, thus it has the ability to extract accurately topography height information [MAC 205], [RED 2004]-[RED 2005].

1. InSAR Topographic Mapping

Interferometric SAR techniques combine two complex SAR images \( I_1 \) and \( I_2 \) to form both the complex interferogram and the coherence map that provides fundamental inquiries on the exploitability of SAR interferograms [ROD 1992]. Prior to interferogram generation, the two SAR images are coregistrated on subpixel accuracy and resampled. Then, interferogram production is achieved by multiplying the first image by the complex conjugate of the second one, and the coherence image is given by the corresponding normalized complex correlation. Thus, the interferogram measures the phase difference between \( I_1 \) and \( I_2 \) (modulo \( 2\pi \)), which is directly related to the path length difference \( \delta r \) as follows:

\[
\phi = k_0, \delta R
\]

(1)

where \( k_0 = \frac{2\pi}{\lambda} \) is the wavenumber; \( \delta R = 2\delta r \) (for repeat pass), and \( \lambda \) is the radar signal wavelength. Terrain elevation is then extracted from the phase difference equation, which depends on orbital parameters and particularly on the interferometric baseline. So, the conversion from the phase difference to surface heights would depend on the InSAR system as follows.
For a single-pass aircraft, the baseline formed by two physical antennas $A_1$ and $A_2$ is known with high precision so that the surface topography is deduced from the interferogram phase using an exact inverse algorithm. The equations of the surface height as a function of orbital parameters are defined by [ZEB 1992] and [ZEB 1994] using the interferometer imaging geometry as shown in Fig. 1(a).

\[
\sin(\theta - \alpha) = \frac{(r + \delta r)^2 - r^2 - B^2}{2rB}
\]  

(2)

\[
z(x) = H - r \cos \theta
\]

(3)

Thus, we measure the phase at each point in an image and apply (1), (2), and (3) to produce the topographic height at each point.

Differentiation of (1-3) yields the error in height estimate as a function of the error in phase estimate:

\[
\sigma_z = \frac{\lambda r}{2 \pi b} \sin \alpha - \cos \alpha \tan(\alpha - \theta) \sigma_\phi
\]

(4)

where $\sigma_z$ and $\sigma_\phi$ are the standard deviation of height and phase respectively.

For repeat-pass interferometric technique, an accurate knowledge of the separated orbit positions allows a fringe pattern estimation of a reference surface and a conversion from phase differences to surface heights. In general, there are uncertainties in the available orbit ephemeris information that may noticeably distort the imaging geometry. In order to obtain an analytical estimation of the required orbit accuracy for InSAR applications, a good understanding of the interferogram phase dependency from the SAR sensor and the object position is required. Using a second-order development, Lin et al [LIN 1991] derived a geometrical formulation that yielded an explicit expression for the path length difference that is a sum of two terms, the first one, $\delta r_1$ associated with the flat-earth surface, and the second one, $\delta r_2$ associated with the elevation. The expression is described by the following approximation:

\[
\delta r = \delta r_1 + \delta r_2 = \frac{Xb}{\sqrt{X^2 + H^2}} + \frac{zHbH}{\sqrt{X^2 + H^2}}
\]

(5)

where $X$ is the distance of the target from nadir (projection of the sensor position on the earth), $\delta r_1$ corresponds to the path length difference and $\delta r_2$ is generated in the interferogram topographic fringes. The rate of the two path length difference gives

\[
\frac{\delta r_2}{\delta r_1} = \frac{zH}{X^2 + H^2}
\]

(6)

2. Geolocation Equations

In our analysis, we consider the airborne InSAR imaging geometry depicted in Fig.2.

The coordinate system denoted by $\hat{X}\hat{Y}\hat{Z}$ is the ECR (Earth Centered Rotating) coordinate system with the origin located at the earth's gravitational center, with $\hat{Y}$ parallel to the nominal track $A_i$, the slant range vector from antenna $A_i$ to target $P$ is given by $\vec{r}_i$, where the subscript $i$ refers to the antenna number or track number. The angle $\theta$, $\beta$ respectively represents the look angle and squint angle of antenna $A_1$ and $\theta_0$ is the look angle to a point on constant height reference surface. The target location vector is given by vector $\vec{P}$, the baseline vector is defined as $\vec{b} = \vec{A}_2 - \vec{A}_1$.

\[
P - \hat{A}_i = \vec{r}_i
\]

(7)

\[
\frac{\lambda f_D}{2} = \langle \vec{r}_i, \vec{v} \rangle
\]

(8)

\[
\phi = \frac{2\pi Q}{\lambda} (\vec{r}_2 - \vec{r}_1)
\]

(9)

where $Q = 1$ for the standard mode InSAR system, $Q = 2$ for the ping-pong mode and $\hat{r}_i$ is the unit vector in the line-of-sight direction of $\vec{r}_i$.

Topography reconstruction by InSAR refers to determine target's 3D location using principal measurements and the imaging geometry of system; it
may be achieved by solving the equation set (7-9). However, the equation set is difficult to be solved directly, that due to its highly nonlinear characteristics. To solve this problem, several reconstruction algorithms have been proposed and they are generally divided into two classes: numerical algorithms [GUT 2000] and analytical algorithms [GOB 1997],[WIL 1997]. In numerical algorithms, the linearization of the nonlinear equation set may be achieved by Taylor expanding InSAR equations (7-9) to the first order and a linear equations system is obtained. Then the unknown position coordinates may be easily achieved by solving the new equation set. The main drawback of numerical algorithms is their inaccuracy due to using only the constant and the linear term of Taylor series. However, analytical algorithms may give more accurate solution than numerical ones because they determine the position coordinates by solving the equation set formed by (7-9) without introducing any approximation. In analytical algorithms, two research trends are resorted to solve the highly nonlinear equation set. Thus, analytical algorithms fall into two classes: the direct analytical method [GOB 1997] and the look vector's orthogonal decomposition method [MAD 1993].

According to (1), the equation relating the scatterer position \( \hat{P} \), a reference position of the platform \( \hat{A}_1 \) and the look vector \( \hat{r}_1 \) is:

\[
\hat{P} = \hat{A}_1 + \hat{r}_1 = \hat{A}_1 + r_1 \hat{r}_1
\]

(10)

where the antenna vector \( \hat{A}_1 \) may be provided by navigation equipments and the slant range \( r_1 \) may be determined according to the principle of radar ranging. Therefore, the determination of the target location is reduced to the determination of the unit vector. Then, look vector's orthogonal decomposition method is introduced just from this idea. The look vector is projected onto an orthogonal basis that is defined as local antenna coordinate system, and then transformed into the general coordinate system. Thus, substituting the unit vector in the general coordinate system into (10) may achieve topography reconstruction.

According to the idea of the orthogonal decomposition, Madsen has introduced a local coordinate system defined on the aircraft and has given a method for the orthogonal decomposition of the look vector [MAD 1993]. However, the look vectors for the two acquisitions have been assumed to be parallel, and thus the range difference approximately equals to the projection of the baseline vector on the look direction. That is to say, a plane wave model of the electromagnetic wave front has been introduced. The simplified model may be valid only for spaceborne InSAR geometries with small swaths and small ratios of the baseline to the slant range [SMA 1996]. However, the airborne InSAR has a relatively larger baseline-range ratio than the spaceborne one. Thus, the plane wave model will introduce a significant height reconstruction error.

### 3. Plane wave model

According to (3), the interferometric phase of InSAR is proportional to the range difference and may be expressed by the baseline vector and range vector as [ROS 2000]

\[
\phi = \frac{2\pi Q}{\lambda} (r_2 - r_1) = \frac{2\pi Q}{\lambda} \left[ -2b < \hat{r}_1, \hat{b} > + \left( \frac{b}{\eta} \right)^2 \right] - 1
\]

(11)

The unit vectors of \( \hat{r}_1 \) and \( \hat{b} \) respectively denoted by \( \hat{r}_1 \) and \( \hat{b} \). \( x = \frac{b}{r_1}, \ \gamma = \frac{2\pi Q r_1}{\lambda}, \ \kappa = 2 < \hat{r}_1, \hat{b} > \). Taylor expanding of (11) at \( x = 0 \) is given by

\[
\phi \approx \frac{\gamma \kappa}{2} x + \frac{\gamma^2}{2} x^2 + \frac{\gamma^3 \kappa}{2(y+1)^2} x^3 + \ldots
\]

(12)

if \( b \ll r_1 \), the following relation can be obtained only by using the constant and linear term of Taylor series

\[
\phi_{pw} \approx \frac{\gamma \kappa}{2} x = \frac{2\pi Q}{\lambda} < \hat{r}_1, \hat{b} >
\]

(13)

where \( \phi_{pw} \) is the plane wave model of the interferometric phase [HAG 1993], [ROD 1992],[ROS 2000], [ZEB 1986]

As discussed above, the target position is determined by the intersection of three surfaces described by (7-9). Fig.3 depicts the range sphere, the Doppler cone and the phase cone relevant to antenna \( A_1 \). The range sphere is centered at antenna \( A_1 \) and has a radius equal to slant range \( r_1 \). The Doppler cone has a generating axis along the velocity vector and the cone angle is proportional to the Doppler frequency. From (11), the phase surface is a hyperboloid with focuses located at \( A_1 \) and a symmetrical axis along the baseline vector. Thus, the target position is given by the intersection locus by the three surfaces. Under the plane wave model, the interferometric phase hyperboloid reduces to a phase cone, and the target position is determined by the intersection locus of range sphere, the Doppler cone and the phase cone, illustrated by \( P \) in Fig.3. According to this interpretation, the plane wave model may introduce an unavoidable reconstruction error.
Physically, the so called plane wave model is that when \( b << r_1 \) is valid. The signals scattered by the same point \( P \) on the surface of topography arrive at the antennas in a parallel manner, thus, the wave front may be approximated as a plane. So, the range difference from antennas to point \( P \) approximately equals the projection of the baseline vector onto the look direction, where \( \Delta r = A_1 C = \hat{r}_1 \cdot \hat{b} \) as illustrated in Fig.4. However, the wave front is actually spherical, the range difference is \( A_1 B \). Thus, an interferometric phase analytic error will be unavoidably introduced. From (11) and (12), this error can be expressed as

\[
\Delta \varphi = \varphi - \varphi_{pw} = \frac{2 \pi Q}{\lambda} (r_2 - r_1) = \frac{2 \pi Q}{\lambda} r_1. \\
\left[ 1 - \frac{2 b \cdot \hat{r}_1 \cdot \hat{b}}{r_1} + \left( \frac{b}{r_1} \right)^2 \right]^{1/2} - 1 + \frac{b}{r_1} \hat{r}_1 \cdot \hat{b} > 0
\]

(14)

As illustrated in Fig.5, for height measurement by InSAR, the universal geometry may be the two-dimensional (2D) InSAR geometry, under which the baseline component lies in the plane of the look vector and the nadir direction, normal to the flight direction. According to this geometry, we have

\[
\hat{r}_1 \cdot \hat{b} = \sin(\theta - \alpha)
\]

(15)

where \( \alpha \) is the angle the baseline makes with respect to a reference horizontal plane. From (14), we obtain the height reconstruction error introduced by the plane wave model

\[
\Delta P_z = -\frac{\lambda r_1}{2 \pi Q \cos(\theta - \alpha)} \left[ 1 + \frac{\lambda \varphi}{2 \pi Q r_1} \right] \sin(\theta) \Delta \varphi
\]

(16)

As mentioned above, the plane wave model was originally applied to the spaceborne InSAR system with a small swath [LI 1990]. The model allows simplified estimation of the baseline based on tie points (points of known elevation on the ground) distributed in range and in azimuth if there is divergence of the baseline. However, one should be aware that significant systematic reconstruction errors are introduced. The plane wave model is inaccurate for satellite geometries with large swaths, and will introduce a systematic height reconstruction error. Moreover, under 2D InSAR, the systematic errors increase in cases with larger track divergences. Although these errors may be mitigated through tie point baseline tweaking [SMA 1993]-[SMA96], they can’t be eliminated completely. Therefore, the model must be carefully treated to get an accurate topography reconstruction by spaceborne InSAR.

For airborne InSAR geometries, the phase error and the height reconstruction error introduced by the plane wave model are approximately of the same order as the accuracies of the phase measurement and the height reconstruction by InSAR [MRS 1996]. On the other hand, even for the airborne InSAR with dual antennas constructed on the rigid platform, the attitude of the aircraft may be dynamic. So, track divergences also exist in airborne InSAR as that in spaceborne InSAR, which in turn increases the height reconstruction error introduced by the plane wave model.

4. Madsen’s Orthogonal Decomposition

In 3D InSAR imaging geometry, the baseline comprises a component along track and component across track. These two components correspond to two different types of applications: the differential interferometry and the topography measurement. On this decomposition basis,
Madsen has introduced an orthogonal decomposition that defines a local coordinate system. The MMC (Madsen Moving Coordinate) system \( \hat{\nu} \hat{v} \hat{w} \) is illustrated in Fig.6, with \( \hat{X} \) parallel to the antenna velocity vector \( \hat{v} \). Its three orthogonal bases are

\[
\hat{v} = \hat{v} \\
\hat{n} = \frac{\hat{b} - \langle \hat{b}, \hat{v} \rangle \hat{v}}{\| \hat{b} - \langle \hat{b}, \hat{v} \rangle \hat{v} \|} \\
\hat{w} = \hat{n} \times \hat{v}
\]  

(17)

Then, the unit look vector \( \hat{r}_1 \) may be expressed by the orthogonal basis as

\[
\hat{r}_1 = \mu_1 \hat{v} + \eta_1 \hat{n} + \zeta_1 \hat{w},
\]

(18)

Under the plane wave model, the interferometric phase now reads [ROS 2000]

\[
\varphi = -\frac{2\pi Q}{\lambda} \langle \hat{r}_1, \hat{b} \rangle
\]

(19)

Substituting (8) and (19) into (18) derives the unit vector in the local system

\[
\hat{r}_{\text{lwvw}} = \left[ \frac{\lambda f_D}{2v} - \frac{\lambda \varphi}{2\pi Q b_n} - \frac{\lambda f_D}{2v} \pm \sqrt{1 - \mu_1^2 - \eta_1^2} \right]
\]

(20)

where \( b_n = \langle \hat{b}, \hat{n} \rangle, \hat{b} = b_n \hat{v} + b_n \hat{n} \). The sign of \( \zeta_1 \) is given by "+" for the left looking of radar and "+" for the right.

5. Reconstruction Algorithm

The transformation from the local coordinate system to the general one can be achieved by the following equation

\[
\Gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}
\]

(21)

Theoretically, after transformation by (21), we have fulfilled the transformation. However, in airborne InSAR, the variation of aircraft attitude (depicted by yaw, pitch and roll) causes the unit look vector in the general coordinates to be transformed into a new coordinate. Thus, in order to achieve the topography 3D reconstruction, the unit look vector must be transformed back into the general coordinate system from the new one again. The transformation can be fulfilled by Euler rotation matrices corresponding to three attitude angles. The above transformation yields the unit look vector in the general coordinate system including the attitude of the aircraft.

\[
\hat{r}_1 = YPR \hat{r}_{\text{lwvw}}
\]

(22)

Where

\[
R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_r & -\sin \theta_r \\ 0 & \sin \theta_r & \cos \theta_r \end{bmatrix},
\]

\[
P = \begin{bmatrix} \cos \theta_p & 0 & -\sin \theta_p \\ 0 & 1 & 0 \\ \sin \theta_p & 0 & \cos \theta_p \end{bmatrix},
\]

\[
Y = \begin{bmatrix} \cos \theta_y & \sin \theta_y & 0 \\ -\sin \theta_y & \cos \theta_y & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

(23)

where \( Y, P, R \) are Euler rotation matrices and \( \theta_r, \theta_p, \theta_y \) are the attitude angles respectively for roll, pitch and yaw.

Finally, we calculate the \((x, y, z)\) coordinates of the targets by using (20) - (23) with (18) and (10).
Fig. 7 show that the Doppler centroid estimation depends strongly on the yaw and pitch. So any errors on the aircraft position are directly translated to the reconstruction location in the same direction.

6. Conclusion

In this paper, the problem of geolocation for interferometric synthetic aperture radar has been presented. The solution is based on knowledge of orbit parameters as well as the range time/Doppler frequency information of each SAR image and their interferometric phase. A topography reconstruction algorithm based on Madsen’s orthogonal decomposition has been analyzed.

The plane-wave approximation is introduced in the derivation of the algorithm. As a consequence, significant errors can be introduced depending upon the choice of method and Earth model especially for the airborne InSAR.

REFERENCES


