Direction of Arrival Estimation Using the Extended Kalman Filter

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Abstract: The technique for estimating the parameters of multiple waves provides a convenient tool for the analysis of multiple-waves fields and eventually for actual applications to mobile communications. Based on the extended Kalman filter (EKF), we analyze a recursive procedure for 2-dimensional directions of arrival (DOA) estimation and we will employ the two-L-shape arrays. A new space-variable model which we call a spatial state equation is presented using array element locations and incident angles. In this paper we briefly recapitulate the most important features of the extended Kalman filter (EKF). The performance of the proposed approach is examined by a simulation study with three signals model. The simulation results show a good estimate performance.

Key words: elevation angles, azimuth angles, spatial state equation, extended Kalman filter.

INTRODUCTION

The authors have proposed to use the Kalman filter technique for adaptive antennas [Greg 04], and showed the advantage of fast convergence through simulation and the principle of initial parameter selection.

The problem of estimating the two-dimensional directions of arrival (DOAs), namely, the azimuth and elevation angles, of multiple sources was the topic of several researches [Krekel 89- Wu 03- Li 91- Liu 05].

We will employ the two-L-shape arrays that showed [Tayem 05] better performances than the one L-shape [Hua 91] and the parallel shape arrays [Marcos 93].

In this paper the detection and estimation of the parameters of multiple waves are discussed.

Being important in radar and communications systems [Kikuma 90], the technique for estimating the parameters of multiple waves is currently the subject of extensive investigation. We will present a method for estimating the elevation and azimuth arriving waves based on the extended Kalman filter [Jinkuan 95].

The general characteristics of a Kalman filter such as most-likelihood and convergence have been discussed in several papers [Shu 03]. On the contrary, we derive a suitable space-variable model in the present paper which we call a spatial state equation using array element locations and incident angles. Simulation results show the performance of our method even at low SNRs.

The rest of the paper is organized as follows: The data model is presented in section 1, the new spatial state equation for incident waves is developed in section 2, in section 3 the extended kalman filter is recapitulated, section 4 presents the 2-D direction of arrival estimation algorithm, section 5 shows simulation results and section 6 makes conclusions.

1. Data Model

Consider the two-L-shape uniform linear array (ULAs) in the x-z and the y-z planes shown in fig.1 with inter-element equals d, using three array elements placed on the x, y and z axes. Each linear array consists of N elements. The element placed at the origin is common for referencing purposes.

Suppose that there are K narrow band sources, S(t), with the same wavelength λ impinging on the array, such that kth source has an elevation angle θk and an azimuth angle φk, k=1, ..., K.

We put the complex base-band representation of
the signal received by the \(n^{th}\) element of one subarray as \(x_n(t)\) \((n = 1, 2, \ldots N)\), the signal sources are far apart from the subarray.

The subarray output vector at the snapshot \(t\) is then given by:

\[
x(t) = (x_1(t), x_2(t), \ldots, x_N(t))^T
\]

\[
= \sum_{k=1}^{K} a(\theta_k, \phi_k) s_k(t) \quad (1), t=1, \ldots, P
\]

“\(T\)” is the transpose, and \(a(\theta_k, \phi_k)\) is the steering vector defined by :

\[
a(\theta_k, \phi_k) = [(1, e^{-j\phi_1}, \ldots, e^{-j\phi_k(N-1)})]^T \quad (2)
\]

with \(\phi_k\) depends on the position of the subarray.

The received signal is given by:

\[
y(t) = Hx(t) + \eta(t) \quad (3)
\]

Where \(\eta\) is an \(N\)-dimensional complex white noise vector with mean zero and covariance \(\sigma I\), \(I\) is the identity matrix of size \(N\).

\[
H = [1 \ldots 1] \quad (4)
\]

with \(K\)-components.

At the \((n+1)^{th}\) element position, the incident wave vector is derived from that at the \(n^{th}\) element position by :

\[
x(n+1) = A(\theta, \phi) x(n) \quad (5)
\]

Where

\[
\theta = [\theta_1, \theta_2, \ldots \theta_K], \quad \phi = [\phi_1, \phi_2, \ldots \phi_K]
\]

\[
A(\theta, \phi) = \begin{bmatrix}
\exp(j\phi) & 0 \\
& \ldots \\
0 & \exp(j\phi_K)
\end{bmatrix}
\quad (6)
\]

2. A new spatial state equation for incident waves

Equation (5) could be called a spatial state equation. Correspondingly, (3) is called a measurement equation. The element of \(X, A\) and \(y\) are all complex in general.

We reformulate the problem involving complex quantities in terms of real quantities, in order to carry out the parameter estimation of incident waves. Let the real and the imaginary parts of \(x_{1,n}\) be denoted by \(z_1\) and \(z_2\) respectively, and the real and imaginary parts of \(\exp(-j\phi_1)\) be denoted by \(\alpha_1\) and \(\alpha_2\), respectively. Continuing in this manner we have

\[
x_{1,n} = z_{2k-1,n} + jz_{2k,n} \quad (7)
\]

\[
\exp(-j\phi_k) = \alpha_{2k-1} + j\alpha_{2k} \quad (8)
\]

It follows that the \(L\)-component complex vector \(X(n)\) can be completely represented by the following real vector \(X_r(n)\) with 2L-components

\[
X_r(n) = [z_{1,n} \ z_{2,n} \ldots z_{2k-1,n} \ z_{2k,n}]
\]

By this way, we can rewrite (5) in terms of real vector as:

\[
X_r(n+1) = A_r(\alpha) X_r(n) \quad (9)
\]
Where

\[
A(\alpha) = \begin{bmatrix}
\alpha_1 & -\alpha_2 & 0 \\
\alpha_2 & -\alpha_1 & 0 \\
\vdots & \vdots & \vdots \\
\alpha_{2k-1} & -\alpha_{2k} & 0 \\
0 & \alpha_{2k} & \alpha_{2k-1} & \ldots
\end{bmatrix}
\]  \hspace{1cm} (10)

\[\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_{2K}], \text{ } k=1,2,\ldots,K\]

The \( A(\alpha) \) is a \( 2L \times 2L \) square matrix, correspondingly, (3) can be rewritten as:

\[Y_r(n) = H_r(n)X_r(n) + N_r(n)\]  \hspace{1cm} (11)

Where

\[Y_r(n) = [\text{Re}(y(n)) \quad \text{Im}(y(n))]^T\]  \hspace{1cm} (12)

\[H_r = \begin{bmatrix}
1 & 0 & 1 & 0 & \ldots & 1 & 0 \\
0 & 1 & 0 & 1 & \ldots & 0 & 1
\end{bmatrix}\]  \hspace{1cm} (13)

\[N_r(n) = [\text{Re}(\eta(n)) \quad \text{Im}(\eta(n))]^T\]  \hspace{1cm} (14)

The size of \( H_r \) is \( 2 \times 2L \), and \( \text{Re}(a) \) and \( \text{Im}(a) \) in \( (14) \) represent the real and imaginary parts of a complex variable \( a \).

3. The Extended Kalman Filter

The Kalman filtering approach for a nonlinear system is based on the first order linearization of a nonlinear state equation using a previous estimate as the centre of the updated linear Taylor approximation. The nonlinear system is given by

\[\dot{\xi}(t + 1) = f(t, \xi(t)) + W(t)\]  \hspace{1cm} (15)

\[V(t) = h(t, \xi(t)) + \eta(t)\]  \hspace{1cm} (16)

\( \xi(t) \) : a state vector, \( V(t) \): measurement vector, \( W(t) \): process noise, \( \eta(t) \): measurement noise, and \( f(.) \) and

\( h(.) \) are nonlinear matrix functions. We will assume \( f(.) \) et \( h(.) \) are differentiable. \( W(t) \) and \( \eta(t) \) are vectors with white Gaussian random elements of zero mean, and their covariance matrices are as follows:

\[\text{Cov}(W(t)W(t)^T) = Q\]  \hspace{1cm} (17)

\[\text{Cov}(\eta(t)\eta(t)^T) = R\]

The extended Kalman filter is described by the following equations:

\[\hat{\xi}(t + 1 / t) = f(t, \hat{\xi}(t))\]  \hspace{1cm} (18)

\[\dot{\xi}(t / t - 1) = \hat{\xi}(t / t - 1) + K(t)[V(t) - h(t, \hat{\xi}(t))]\]  \hspace{1cm} (19)

\[K(t) = P(t / t - 1)\prod(t, \hat{\xi}(t / t - 1))^T \left[ \prod(t, \hat{\xi}(t / t - 1))P(t / t - 1)\prod(t, \hat{\xi}(t / t - 1)) + R \right]^{-1}\]  \hspace{1cm} (20)

\[P(t / t) = P(t / t - 1) - K(t)\prod(t, \hat{\xi}(t / t - 1)P(t / t - 1))\]  \hspace{1cm} (21)

\[P(t / t + 1) = F(t, \hat{\xi}(t / t))P(t / t)F(t, \hat{\xi}(t / t))^T + Q\]  \hspace{1cm} (22)

Where \( \hat{\xi}(t / t) \) and \( \hat{\xi}(t / t - 1) \) denote the conditional mean estimates of \( \hat{\xi}(t) \), based on the measurements \( V(t) \) and \( V(t-1) \), respectively. \( F(t, \hat{\xi}(t / t)) \) and \( \prod(t, \hat{\xi}(t / t - 1)) \) denote the following Jacobean matrices evaluated at the values of \( \hat{\xi}(t / t) \) and \( \hat{\xi}(t / t - 1) \), respectively.

\[
F(k, \hat{\xi}(k / k)) = \frac{\partial}{\partial \xi} f(k, \xi(k)) \bigg|_{\xi = \hat{\xi}}
\]  \hspace{1cm} (23)

\[
\prod(k, \hat{\xi}(k / k - 1)) = \frac{\partial}{\partial \xi} h(k, \xi(k)) \bigg|_{\xi = \hat{\xi}}
\]  \hspace{1cm} (24)

\( K(t) \) denotes the Kalman gain matrix. \( P(t / t) \) and
$P(t/t-1)$ denote the estimation error covariance matrices.

The above extended Kalman filter can also be used to estimate unknown parameters in a linear system. In the following, we apply it to the problem where a vector $\alpha$ is given by:

$$X_k(t + 1) = \Lambda(\alpha)X_k(t) + W(t) \quad (25)$$

$$Y_k(t + 1) = C(\alpha)X_k(t) + \eta(t) \quad (26)$$

The matrices $\Lambda(\alpha)$ and $C(\alpha)$ are of finite dimensions. It is assumed that the matrix elements are differentiable with respect to $\alpha$. $W(t)$ and $\eta(t)$ are independent of $\alpha$.

The extended Kalman filter to determine the unknown parameter vector $\alpha$ now can be obtained by extending the state vector $X_h$ to the vector $\xi(k)$ by including the parameter vector $\alpha = \alpha(k)$ which depends on the parameter $k$,

$$\xi(t) = \begin{bmatrix} X_k(t) \\ \alpha(t) \end{bmatrix} \quad (27)$$

Equations (25) and (26) are rewritten as follows:

$$\xi(t + 1) = f(t, \xi(t)) + \begin{bmatrix} W(t) \\ 0 \end{bmatrix} \quad (28)$$

$$Y_k(t) = h(t, \xi(t)) + \eta(t) \quad (29)$$

Where

$$f(t, \xi(t)) = \begin{bmatrix} \Lambda(\alpha)X_k(t) \\ \alpha \end{bmatrix} \quad (30)$$

$$h(t, \xi(t)) = C(\alpha)X_k(t) \quad (31)$$

Because (28) and (29) have the same forms as (15) and (16), the extended Kalman filter in (18)-(22) can be applied to (28) and (29). It shows that the parameter vector $\alpha$ can be estimated by the extended Kalman filter.

4. The 2-D direction of arrival estimation algorithm

4.1. Estimation of the elevation angle

Let $v(t)$ be the $N x 1$ signal received at the linear subarray in the z axes at the snapshot $t$.

$$v(t) = (v_1(t), v_2(t), \ldots, v_N(t))^T = \sum_{k=1}^{K} a(\theta_k)s_k(t) + \eta_{kz}(t), t=1,\ldots,P \quad (32)$$

Where

$$a(\theta_k) = [1, e^{-j\phi_1}, \ldots, e^{-j\phi_{N-1}}]^T \quad (33)$$

and $\phi_{k,z} = \frac{2\pi(n-1)d}{\lambda}$

$\theta_k$ is the elevation angle of the $k^{th}$ source signal. $\eta_{kz}(t)$ is the additive White Gaussian noise of the $k^{th}$ source signal at the snapshot $t$.

4.2. Estimation of the azimuth angle

Let $x(t)$ be the $N x 1$ signal received at the linear subarray in the x axes at the snapshot $t$.

$$x(t) = (x_1(t), x_2(t), \ldots, x_N(t))^T = \sum_{k=1}^{K} a(\theta_k, \phi_k)s_k(t) + \eta_{kx}(t) \quad (35)$$

Where

$$a(\theta_k, \phi_k) = [1, e^{-j\phi_1}, \ldots, e^{-j\phi_{N-1}}]^T \quad (36)$$

and $\phi_{k,x} = \frac{2\pi(n-1)d}{\lambda}$

$\phi_k$ is the azimuth angle of the $k^{th}$ source signal. $\eta_{kx}(t)$ is the additive White Gaussian noise of the $k^{th}$ source signal at the snapshot $t$ in the x subarray with mean zero and covariance $\sigma^2 I$.

In the same way we estimate the azimuth angle.
by $H_r$ and $Y_r(m)$ in (11), respectively.

2) Extending the state vector by including the unknown incident angles in the state vector. It is obtained from (2), (7) and (27) as follows:

$$\dot{\xi}(n) = [z_{1,n} \ z_{2,n} \ ... \ z_{K,n}]^{T} \alpha_1 \ \alpha_2 \ \alpha_3]^{T} \ (42)$$

3) Adapting the extended state vector (42) to (28) and (29), to obtain the nonlinear state equation.

4) Using the nonlinear state equation, the extended Kalman filter (18) . . . (22) is carried out to obtain the estimation of the elevation or the azimuth arrival waves.

5. Simulation Results

Computer simulations have been conducted to evaluate the 2-D DOA estimation performance of the proposed method. The parameters used in the simulation are as follows:

The sensors displacement $d$ is taken to be half the wave length of the signal waves. The incident waves are three waves $K=3$, with directions of arrival DOA $(\theta_k, \phi_k)$, $k=1...K$. The additive noise is white Gaussian processes. The $L=250$ is the number of snapshots per trial.

All the waves are assumed to be direct waves and the power levels are a constant 2, 3.2 and 3, respectively. The measurement noise $\eta(t)$ in (16) is assumed to be 0.1. The process noise $W(t)$ is assumed to be zero as the state (15) represents the elements state at the same time. The convergence of estimated values of the elevation and azimuth angles are plotted in fig.2 and fig.3, respectively. The abscissa is the number of iterations, and means the numbers of necessary elements.

In figures.2 (a) and (b) the waves are located at $(60^\circ,20^\circ)$, $(70^\circ,30^\circ)$ and $(80^\circ,40^\circ)$, respectively and signal to noise ratio (SNR)=10 dB. This is an example of the most common case because the incident angles differences are medium and, accordingly, the convergence is not bad. It is shown that the estimated values steadily converge to the actual values. About 110 iterations are required to achieve a steady state for the elevation angles. In other words, 110 out of 250 elements for the z axes array are used for convergence. For the azimuth angles, only 85 iterations are required to obtain the convergence to the real azimuth angles.

In figures.3 (a) and (b), three waves are located at $(70^\circ,20^\circ)$, $(74^\circ,24^\circ)$ and $(78^\circ,28^\circ)$, respectively. Despite the proximity of the elevation and azimuth angles, the convergence characteristics of angles estimations show no degradation from fig.2. Although the variations of the elevation angles are larger and the convergence speeds are slower than in fig.2, the estimates errors are quite small after achieving the convergence state after 180 iterations.
Figures 4 (a) and (b) show the histogram plots for the joint elevation and azimuth angles, respectively, for a single source with DOA located at $(60^\circ,20^\circ)$ and signal to noise ratio (SNR) = $10$ dB, by using the extended kalman filter of the 2-L shape arrays. We observe that the method gives a very close joint DOA estimation and the clear peaks appear around $(60^\circ, 20^\circ)$. No failure can be observed in the estimation of the 2-D directions of arrival, but it is clear that the proposed algorithm improves the performance significantly and it also reduces the estimation error of both the azimuth and elevation angles.

Figure 2: Convergence results for the sources located at $(60^\circ,20^\circ)$, $(70^\circ,30^\circ)$ and $(80^\circ,40^\circ)$, respectively. SNR = $10$dB. (a) Elevation angles. (b) Azimuth angles.

Figure 3: Convergence results for the sources located at $(70^\circ,20^\circ)$, $(74^\circ,24^\circ)$ and $(78^\circ,28^\circ)$, respectively. SNR = $10$dB. (a) Elevation angles. (b) Azimuth angles.
6. CONCLUSIONS:

An antenna array configuration was proposed using the extended kalman filter method, for the 2-D azimuth and elevation angle estimation problem. The results obtained are summarized as follows.

(1) A space variable model, called spatial state equation is derived, using the elements array of the 2-L shape antenna. (2) We show that the extended kalman filter algorithm, which can be applied to all array configurations, is simple and applicable to practical cases. (3) The proposed scheme reduces the estimation error of both the azimuth and elevation angles and it shows a good performance at low SNRs and at the proximity of the waves.

It should be noted that the idea of the proposed approach is different from the well-known MUSIC or the PM algorithms. The angles of the arrival waves are directly estimated from the signal received at each array element, though MUSIC and PM algorithms obtain the estimation based on the spectral analysis techniques.

The performance of the proposed approach is not affected by the change of the incident waves during the computing time of the algorithm, because the signals at each array element are collected at the same sampling time.

REFERENCES:


