Chaotic Signal Generation from a Switched Dynamical System

Fatima El Guezar *, Hassane Bouzahir** and Pascal Acco *

*Laboratoire d’Etude des Systèmes Informatiques et Automatiques (LESIA), INSA 135, Avenue de Rangueil-31077, Toulouse Cedex 4, France
fatima.el_guezar@insa-toulouse.fr
pascal.acco@insa-toulouse.fr

**Equipe Signaux Systèmes et Informatique (ESSI), BP 1136 ENSA, Agadir, 80000 Morocco
hbouzahir@yahoo.fr

Abstract: In this paper, we explore some modeling capabilities of Scicos. Our aim is to generate chaos from a simple hybrid dynamical system. We give the chaotic dynamics of the voltage-mode controlled buck converter circuit in open loop as a case study. By considering the voltage input as a bifurcation parameter, we observe that the obtained Scicos simulations show that the buck converter is prone to subharmonic behavior and chaos. We also present the corresponding bifurcation diagram.

Key words: chaotic signal, hybrid system, switched dynamical system, Scicos-Scilab.

INTRODUCTION

Hybrid dynamical systems (HDS) have attracted considerable attention in recent years. HDS arise from the interaction between continuous variable systems (i.e., systems that can be described by a difference or differential equation) and discrete event systems (i.e., systems where the state transitions are initiated by events that occur at discrete time instants).

Switched piecewise linear systems are an important class of hybrid systems that are simple and can have very rich and typical nonlinear dynamics such as bifurcations and chaos.

As example, DC–DC switching converters are switched piecewise linear systems [1]. The three basic power electronic converters buck, boost and buck-boost are variable structure systems that are highly nonlinear. The kind of piecewise model may present nonlinear phenomena such as bifurcations and chaos.

The study of nonlinear dynamics of DC-DC converters started in 1984 by Brockett’s and Wood’s research [2]. Since then, chaos and nonlinear phenomena in power electronic circuits have stolen the spotlight and have attracted the attention of different research groups. Different nonlinear phenomena were investigated such as flip bifurcation or period doubling and its related route to chaos [3-11] or quasiperiodicity route to chaos [13, 14], as well as border collision bifurcation [11, 12, 16].

There are many modeling techniques, programming languages, and design toolsets for HDS. To model and simulate our HDS, we use Scicos (Scilab Connected Object Simulator) which is a Scilab package for modeling and simulation of dynamical systems including both continuous and discrete time subsystems [17, 18]. Scilab (Scientific Laboratory) is a scientific software package for numerical computations that provides a powerful open computing environment for engineering and scientific applications [19]. It has been developed at INRIA and ENPC and is freely available for download at “http://www.scilab.org”.

This paper aims to study and analyze some dynamic phenomena that can occur in the voltage-mode controlled buck converter. We also show from Scicos simulations that variation of the voltage input can lead to interesting route to chaos.

In section 2, the general equation of a hybrid dynamical system is briefly recalled. In Section 3, we explain the operation of the voltage-mode controlled buck converter. Then, we introduce the state equations of the circuit in question. In section 4, we comment on the obtained Scicos simulations. We end by some concluding remarks.

1. Hybrid Dynamical System

The evolution of an autonomous hybrid dynamical system can be described by [15]:
\[ \frac{dx}{dt} = f(x(t), q(t)), \quad x(t_0) = x_0, \]
\[ q(t) = e(x(t), q(t)), \quad q(t_f) = i_0, \]

where \( x(t) \in \mathbb{R}^n \) is the continuous state vector and \( q(t) \in Q = \{1, \ldots, n_Q\} \) denotes the discrete state. The state space is \( H = \mathbb{R}^n \times Q \) and the initial state is supposed belonging to the set of initial conditions \( \mathcal{H}_0 \). The function \( e: \mathbb{R}^n \times Q \rightarrow Q \) describes the change of the discrete state. The change from one distinct discrete state to another is called a transition or a switch. A transition between two states \( i \) and \( j \) occurs if \( x(\cdot) \) reaches the switch set \( S_{i,j} \).

Among important classes of hybrid systems, we find piecewise linear systems that are described by:
\[ \frac{dx}{dt} = A(q)x + B(q) \quad \text{with } x(0) = x_0 \]

where \( A(q) \in \mathbb{R}^{n \times n} \) and \( B(q) \in \mathbb{R}^{n} \) are matrices depending on \( q \).

2. Voltage-mode controlled Buck converter

2.1. Operation of voltage-mode controlled Buck converter

A voltage feedback buck converter is represented in Fig.1. This consists of a basic RLC circuit, a diode and a switching element \( S \). The aim of the circuit is to maintain a desired voltage, across the load resistance \( R \), lower than the input voltage \( E \). This can be realized by the help of feedback PWM control. The PWM control of a switched converter is achieved by obtaining a control voltage \( v_{con}(t) \), as a linear combination of the output capacitor voltage \( v_C(t) \) and a reference signal \( V_{ref} \) in the form:
\[ v_{con}(t) = a \left( V_C(t) - V_{ref} \right) \]

where \( a \) is the gain of the error amplifier. The control voltage is compared with an externally generated sawtooth wave \( V_{ramp}(t) \) given by:
\[ V_{ramp}(t) = V_L - \frac{(V_{U} - V_L) t}{T} \quad \text{for } t \in [0,T] \quad (4) \]

The output of his comparator is used to determine the state of the switch \( S \), in such way that:
\( S \) is \( \text{off} \) when \( v_{con}(t) \geq V_{ramp}(t) \) and \( S \) is \( \text{on} \) when \( v_{con}(t) < V_{ramp}(t) \).

2.2. State Equations

When operating in continuous conduction mode (CCM), two switch states can be identified:
- Switch \( \text{off} \) and diode \( \text{on} \)
- Switch \( \text{on} \) and diode \( \text{off} \)

Whether the switch is on or off, the buck converter can always be described as a second order linear system, whose states are the voltage \( v_C \) across the capacitor, and the current \( i_L \) along the inductor.

The general equation that models operation of the buck converter takes the form:
\[ \frac{dx}{dt} = f_q(x) = A(q)x + B(q) \quad \text{with } q \in Q = \{0,1\} \quad (5) \]

For \( q=0 \) and \( q=1 \), we obtain the following two systems of differential equations:
\[ S_{on} : \quad \frac{dx}{dt} = f_0(x) = A_0x + B_0 \]
\[ S_{off} : \quad \frac{dx}{dt} = f_1(x) = A_1x + B_1 \]

where \( x = \begin{bmatrix} v_C \\ i_L \end{bmatrix} \) is the vector of the state variables and \( q \in Q = \{0,1\} \) is the discrete variable.

The \( A \)'s and \( B \)'s matrices and vectors are given by:
\[ A_0 = A_1 = \begin{bmatrix} -1 & 1 \\ \frac{1}{RC} & -1 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{and } B_1 = \begin{bmatrix} 0 \\ E \end{bmatrix} \]

The border function is given by
\[ \beta(x) = v_{con}(t) - V_{ramp}(t) = a V_C(t) - a V_{ref} - V_L - \frac{(V_{U} - V_L) t}{T} \quad \text{for } t \in [0,T] \]

Therefore the switching sections of each subsystem \( S_{on} \) and \( S_{off} \) are given by:
\[ \beta_{off,on} = \{ (x(t) \in \mathbb{R}^2 \times R : \beta(x(t)) \geq 0 \} \]
\[ \beta_{on,off} = \{ (x(t) \in \mathbb{R}^2 \times R : \beta(x(t)) < 0 \} \]

The buck converter in CCM switches between two systems \( S_{on} \) and \( S_{off} \) if the switching sections \( \beta_{on,off} \) and \( \beta_{off,con} \) are reached. Fig. 2 shows the transition diagram.
3. Simulation Results and comments

We choose the parameter values: \(L = 30\, \text{mH}, \quad T = 400\, \mu\text{s}, \quad R = 22\, \Omega, \quad C = 47\, \mu\text{F}, \quad a = 8.4, \quad V_{\text{ref}} = 11.3\, \text{V}, \quad V_L = 3.8\, \text{V} \) and \( V_U = 8.2\, \text{V} \). We consider the input voltage \( E = 20\sim50\, \text{V} \) as parameter of bifurcation. By varying \( E \), the circuit changes its qualitative behavior from a stable periodic system to another system that exhibits chaos.

At first, using Scicos we draw the one-parameter bifurcation diagram given in Fig. 3 where the input voltage \( E \) is the bifurcation parameter and the sampled \( v_C \) is the variable. By increasing \( E \), the displayed diagram shows a period doubling route to chaos.

![Fig. 3. Period doubling bifurcation diagram.](image)

For different increasing values of \( E \), we give the capacitor voltage wave form \( v_C \) and its corresponding phase plan \( v_C-i_L \).

By choosing \( E = 23\, \text{V} \), we get a fundamental periodic operation. This periodic regime is possible just for small values of \( E \). Fig. 4 show the fundamental periodic operation. Fig. 4(a) displays the capacitor voltage wave form; Fig. 4(b) gives the corresponding phase plan.

For \( E = 37.5\, \text{V} \) and \( E = 41.5\, \text{V} \), subharmonic operation has been found. Fig. 5 presents 2-T periodic subharmonic operation, whereas Fig. 6 illustrates 4-T periodic subharmonic operation.

However, the chaotic operation is given for \( E = 48\, \text{V} \). Fig. 7: (a) indicates a chaotic signal with infinite order and (b) the phase plan \( v_C-i_L \) corresponds to a chaotic attractor.

### 4. Conclusion

This article has illustrated a Scicos numerical study of the voltage-mode controlled buck converter that is modeled by a hybrid system. Variations of the voltage input can lead to interesting route to chaos; the system pursues period doubling bifurcation. The displayed simulation results prove also that Scicos is a powerful simulator of hybrid systems.

### 5. References


Fig. 4. Fundamental periodic operation (E=23V): (a) Time waveform of the capacitor voltage (b) Phase plan.

Fig. 5. 2T subharmonic operation (E=37.5V): (a) Time waveform of the capacitor voltage (b) Phase plan.
Fig. 6. 4T subharmonic operation (E=41.5V): (a) Time waveform of the capacitor voltage (b) phase plan.

Fig. 7. Chaotic Regime (E=48V): (a) Time waveform of the capacitor voltage (b) Phase plan.