Impact of Gain Dynamics on the Cross-Polarization Modulation Effect in Semiconductor Optical Amplifier

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Abstract: The object of this paper is to present a qualitative and an exhaustive study of the polarization rotation effect in the Semiconductor Optical Amplifier (SOA) structure that is exploited in high speed optical communication networks to assure various applications based on the nonlinearities that it presents. Particularly, we analyze the impact of the pump polarization change at the input of the structure on its behaviour and the evolution of its characteristic parameters (gain, efficient index, phase shift, state of polarization) at the output. This analysis is based on a method that associates the modes coupling model and the Mueller-Stokes formalism.

Key Words: Semiconductor Optical Amplifier (SOA), High speed, Nonlinearity, Polarization Rotation.

INTRODUCTION

The increased need for high speed optical communication networks is the result, on the one hand, of the growth of the users’ number and on the other hand of the increase of the require bandwidth for the different services and applications. To satisfy these demands in term of speed, it is indispensable to find some pragmatic solutions. Among the widespread solutions, we distinguish the one that has for object to conceive and to make use of high speed communication systems based on Semiconductor Optical Amplifier (SOA) configurations. The last have been considered as promising candidates to provide not only optical signal amplification but also optical signal processing, particularly for high speed nonlinear devices in communication networks.

The SOAs are devices in which the carriers’ density is raised. They are characterized by the multifunctionality, the compactness, the ability of integration and the strong nonlinearities that make them very attractive for the high speed all-optical communication systems. The cross gain modulation (XGM), the cross phase modulation (XPM), the four wave mixing (FWM) and the cross-polarization modulation (XPoLM) are the main physical effects, resulting of nonlinear effects, in SOAs that are exploited to assure high speed devices as the optical 3R regenerators [1], wavelength converters in the WDM networks [2], [3], the all-optical switches [5], the optical logic gates [4], [6], that could be used in the future optical network configurations for identification of heading [7] and the data encoding [8].

In this paper, we are interested to analyze the nonlinear polarization rotation in SOA structure by application of the Mueller-Stokes formalism to a model that takes in account:

- the un-homogeneity of the carriers density distribution along the amplifier middle,
- the coupling between TE (transverse electric) and TM (transverse magnetic) modes,
- the effects of polarization rotation,
- the dependence of the SOA gain on polarization.

1. Analysis Method of nonlinear polarization rotation effect in SOA

1.1. Effect of the Cross-Polarization Modulation

Owing to the asymmetry of the SOA structure [9], the distribution of the carriers’ density along this structure, in saturation regime will be strongly disrupted in presence of a pump signal. This perturbation will take place with a different manner according to the axes of the SOA structure [5], on account of two essential factors: the difference of gain...
and the difference between TE/TM confinement factors that will induce a different saturation of the SOA axes. 

The difference of TE/TM gain is defined as [10]:

\[
\Delta G_{db} = 10 \log \left[ \exp \left( (g_{TE} - g_{TM}) \cdot L \right) \right] 
= 4.343 \left[ (\Gamma_{TE} - \Gamma_{TM}) g_m + (\alpha_{TM} - \alpha_{TE}) \right] \cdot L 
\tag{1}
\]

\(g_{TE}\) and \(g_{TM}\) are gain coefficients, \(\Gamma_{TE}\) and \(\Gamma_{TM}\) are confinement factors, \(\alpha_{TE}\) and \(\alpha_{TM}\) are the efficient losses, respectively for TE and TM modes. \(g_m\) is the gain material coefficient. \(L\) represents the active zone length of the SOA.

The cross-polarization modulation is a phenomenon resulting of a nonlinear effect that appears in the SOA structure whose origin is the carriers’ dynamics. It is explained by the modification of the polarization state of a signal that propagates in the SOA according to the polarization and the strength of a control signal (pump) there injected simultaneously.

While injecting a signal at the input along TE axis or TM axis of the SOA with a linear polarization, then the polarization at the output is going to undergo a change that is bound to the difference of the TE gain and TM gain, to the birefringence and the modification of the SOA waveguide axes.

In order to better characterize this polarization change exhaustively, we propose in the following a modelling method that associates the modes coupling model and the Mueller-Stokes formalism.

### 1.2. Formulation of model equations by Mueller-Stokes Formalism

While referring to the coupled modes equations developed by [11] that take in account the coupling between TE and TM modes, the evolution of the electromagnetic field envelope in the amplifier middle of the SOA structure can be written under the following matrix form:

\[
\frac{\partial A(z)}{\partial z} = jM(A(z))A(z) 
\tag{2}
\]

With:

\[
A(z) = \begin{pmatrix} A_{TE}(z) \\ A_{TM}(z) \end{pmatrix} 
\tag{3}
\]

\[
M(A(z)) = \begin{pmatrix} -j \cdot g_{TE}(z) & -j \cdot K_r \cdot e^{-j\Delta \beta \cdot z} \\ j \cdot K_r \cdot e^{j\Delta \beta \cdot z} & -j \cdot g_{TM}(z) \end{pmatrix} 
\tag{4}
\]

Where:

\[
g_{TE}(z) = \frac{\Gamma_{TE} \cdot g_{m,0}}{1 + \left| A_{TE}(z) \right|^2 + \left| A_{TM}(z) \right|^2} E_0^{-2} - \alpha_{TE} 
\tag{5}
\]

\[
g_{TM}(z) = \frac{\Gamma_{TM} \cdot g_{m,0}}{1 + \left| A_{TE}(z) \right|^2 + \left| A_{TM}(z) \right|^2} E_0^{-2} - \alpha_{TM} 
\tag{6}
\]

\[
\Delta \beta = \beta_{TM} - \beta_{TE} 
\tag{7}
\]

\(K_r\) represents the coupling coefficient, \(\beta_{TE}\) and \(\beta_{TM}\) are the propagation constants, respectively for TE and TM modes.

In order to value the polarization change at the output of the SOA structure in relation to its state at the input, Mueller-Stokes formalism is put in evidence. Stokes parameters \((S_0, S_1, S_2, S_3)\) are defined as follows [12]:

\(S_0\) is a parameter that translates the total intensity.

\(S_1\) refers to the intensities difference between the horizontal polarization and the vertical polarization.

\(S_2\) makes reference to the difference between intensities transmitted by axes (+45°, -45°).

\(S_3\) is a parameter that expresses the difference between intensities transmitted for the left and right circular polarizations.

\[
\begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} A_{TE}^2 + A_{TM}^2 \\ A_{TE}^2 - A_{TM}^2 \\ 2 \cdot A_{TE} \cdot A_{TM} \cdot \cos(\Delta \phi) \\ 2 \cdot A_{TE} \cdot A_{TM} \cdot \sin(\Delta \phi) \end{pmatrix} 
\tag{8}
\]

With:

\[
\Delta \phi = (\phi_{TM} - \phi_{TE}) 
\tag{9}
\]

The signal phase shift \(\Delta \phi\) at the SOA output makes reference to the delay between TE and TM components of the electric field, which is explained by the difference between phase velocities of TE/TM modes, generated by the presence of the birefringence in the SOA. Thus, it can be expressed by Stokes parameters as:

\[
\Delta \phi = \arctan \left( \frac{S_2}{S_1} \right) 
\tag{10}
\]

The strong un-homogeneity, that has for origin the different saturation of the SOA axes, affects the intrinsic parameters of the middle amplifier considerably and then the density of photons is modified. Therefore, the birefringence will be strongly modified along the SOA structure; as a result, there will be a difference between the efficient indices for TE and TM modes.

The real part of the efficient refractive indices difference is under the form:

\[
[\Delta n]_{Re} = \frac{\lambda_0 \cdot \Delta \phi}{2 \cdot \pi \cdot L} 
\tag{11}
\]

While the imaginary part of the efficient refractive
indices difference is given by:

\[
[\Delta n]_{\text{m}} = -\frac{\gamma_0}{4.\pi L} \ln \left( \frac{P_{TM}}{P_{TE}} \right)
\]  

(12)

Where:

\[
P_{TM} = |A_{TM}|^2 \quad \text{and} \quad P_{TE} = |A_{TE}|^2
\]  

(13)

Stokes parameters presented in (8) can be also rewritten according to the azimuth (polarization angle) noted \( \psi \) and the ellipticity noted \( \chi \), as follows [13]:

\[
\begin{pmatrix}
S_0 \\
S_1 \\
S_2 \\
S_3
\end{pmatrix}
= \begin{pmatrix}
S_0 \\
S_0 \cdot \cos(2\psi) \cdot \cos(2\chi) \\
S_0 \cdot \sin(2\psi) \cdot \cos(2\chi) \\
S_0 \cdot \sin(2\chi)
\end{pmatrix}
\]  

(14)

Therefore, the polarization change in SOA can be analyzed and identified by the azimuth and the ellipticity that can be expressed according to Stokes parameters as follows:

\[
\psi = \frac{1}{2} \arctan \left( \frac{S_2}{S_1} \right)
\]  

(15)

\[
\chi = \frac{1}{2} \arcsin \left( \frac{S_3}{S_0} \right)
\]  

(16)

\[
\begin{array}{c}
\text{Fig.1: Descriptive diagram of the polarization angle (azimuth) noted } \psi \text{ and the ellipticity noted } \chi. \\
\end{array}
\]

1.3. Formulation of the numeric method for the calculation of SOA parameters

In order to value the change of polarization (orientation, ellipticity), the gain distribution, the phase shift evolution and the efficient indices variation at the output of the SOA structure, Stokes parameters must be calculated. For this reason, the differential equations under the matrix form (2) must be solved. For this, we will consider the resolution numerically by a method whose objective is to provide an efficient calculation of the equations numeric integration.

The equation (2) admits an initial solution given by:

\[
A_0(z_0 + \Delta z) = \exp (- j M(z_0) \Delta z) A(z_0)
\]  

(17)

The exponential term can be developed like a finished summons of Taylor series terms, the equation (17) becomes thus:

\[
A_0(z_0 + \Delta z) = \sum_{n=0}^{m} \frac{1}{n!} \left( - j \Delta z M(z_0) \right)^n A(z_0) + O^{m+1}(\Delta z)
\]  

(18)

\( O^{m+1}(\Delta z) \) makes reference to the error to the order \( m+1 \).

Because the matrix \( M \) is not constant in the \( \Delta z \) interval, the application of a first-order correction to the initial solution would be indispensable. This correction is given by:

\[
A_c(z_0 + \Delta z) = \frac{j \Delta z}{2} \left[ M(z_0 + \Delta z) - M(z_0) \right] A(z_0)
\]  

(19)

The final solution has the form as follows:

\[
A(z_0 + \Delta z) = A_0(z_0 + \Delta z) + A_c(z_0 + \Delta z)
\]  

(20)

In the numeric implementation of the method, parameters took in account are: Taylor series is valued until the twentieth order, the SOA structure is massive type based on InGaAsP/InP having the following features: length \( L=100\mu m \), width \( W=3\mu m \), thickness \( d=0,25\mu m \), refractive index \( n=3,7 \), carriers density \( N=2.10^{24} \m^{-3} \), pump signal wavelength \( \lambda_0=1,55\mu m \).

2. Results and Discussion

In the following, we’re interested to analyse the effect of polarization state change at the output following an injection of a signal pump, according to different linear polarizations (quasi-TE, quasi-TM, \( \pi/4 \), \( \pi/3 \)), at the input of the SOA structure.

The evolution of the polarization orientation or azimuth of the signal at the SOA output as a function of the polarization and the pump power is illustrated in figures 2 and 3.

These results show that there will be a remarkable change of the polarization state at the SOA output particularly for low values of the pump. However, this polarization rotation not only varies with the pump signal power but also as a function of its polarization at the input.
The increase of the pump power has for effect the convergence of the polarization angle $\psi$ at the SOA output toward a constant value, very low and near to zero (for a pump on quasi-TE, at $\pi/4$ and at $\pi/3$, $\psi$ tends to -2.8% radian whereas for a pump on quasi-TM, $\psi$ tends to +0.2% radium); who corresponds the tendency of the polarization to be linear again.

Results in Fig.4 and Fig.5 represent the ellipticity evolution as a function of the pump signal power for different polarizations at the input. They show that ellipticity, which is used to describe how the vector wave will be flatten, undergo a big variation in the zone of low values of the pump. As consequence, the polarization rotation becomes elliptic at the output of the SOA structure.

Some values of the pump power correspond to an ellipticity angle that is included in the interval $[0, \pi/2]$; what proves that the signal is polarized elliptically. On the other hand, the increase of pump power, which coincides with the tendency of the ellipticity angle toward the null value, has for effect the passage of the polarization from the elliptic state toward the linear one.

The SOA presents different efficient refractive indices according to its axes. Indeed, this variation of efficient indices depends on the polarization and the pump power injected in the SOA structure. It is shown in Fig.6, Fig.7, Fig.8 and Fig.9. This variation of the birefringence will generate the phase difference variation between TE and TM field components. This TM/TE phase shift is presented as a function of the pump power for different polarizations in Fig.10 and Fig.11.
These results show that the increase of pump power has for effect a strong reduction of the carriers’ density in the SOA active region; therefore a modification of the efficient refractive index will appear. Consequently, a variation of phase shift is gotten. The TE/TM efficient indices perturbation is different because of the difference of the gain between these modes.

The observed variation for low pump powers is not due to the SOA intrinsic birefringence only, but also to the influence of the signal injected in the structure. Indeed, in this zone where the pump power is low, the amplification of the signal is more important and as consequence there is perturbation of the SOA.
We can note that owing to the SOA structure asymmetry, the confinement factors, the efficient refractive index and the carriers’ distribution are not identical to the TE/TM orientations of the SOA. Consequently, a polarization rotation will appears at the output that is bound to the polarization and the pump power.

3. Conclusion

In this letter, we analyzed the impact of the gain on the cross-polarization modulation effect in a Semiconductor Optical Amplifier (SOA) with a method that is based on the Mueller-Stokes formalism and the coupled modes theory.

However, we showed in this study that the linear polarization of a signal injected according to the SOA axes undergoes a rotation and it becomes elliptic at the output of the structure. This polarization rotation is bound to the difference of the TE/TM gain, to the birefringence and to the axes modification of the SOA structure. In fact, it depends on the polarization and the pump power injected in the SOA structure.

Thus, we can announce that three phenomena that intervene in the cross-polarization modulation effect, namely: the induced birefringence, the dependence in gain of the SOA and the coupling between modes.

However, this nonlinear phenomenon is exploitable to achieve high speed all-optical functions such as the optical logic gates, the wavelength converters and the switches.

REFERENCES