Sensitivity Function Loop Shaping Design For An Optical Disc Drive

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Abstract: A new approach for active noise and disturbance rejection for an optical disk drive subject to periodic disturbances with time varying fundamental frequency is presented. We consider the problem of loop shaping of the sensitivity function by using estimators. The proposed method combines pole placement with sensitivity function loop shaping in the frequency domain. The approach is non adaptive and the frequencies are chosen according to the desired shape of the sensitivity function in the desired bandwidth. Damped sine wave models are introduced in the estimator for disturbance tuning reduction, for active damping rejection, and to achieve stability robustness.

Keywords: CD-ROM, disturbance estimator, loop shaping, Multimedia.

INTRODUCTION

During the recent years, the reduction of noises, vibrations and loop shaping in various plants has become a main line of research and is the object of numerous publications. For many multimedia applications, it is desired to achieve a high speed in increasing data rate and reducing access time. Track following problem, for optical disk drive such as CD-ROM, is to control the position of the optical spot in such away that it follows the desired track (within 0,1 µm) of optical disk media which is usually deviated from the concentric circles due to the disk eccentricity. The displacement error caused by this last one amounts to 280 µm in the worst case. The optical disk drive measures the position of the spot by a relative position error between the desired track and the actual position of the spot. Therefore, the disc eccentricity affects this measure as a sinusoidal disturbance whose frequency is the one of the disk spindle motor [POL 92], [WAT 94]. The basic problem for the compact disc mechanism control is the huge variety of sources for disturbances and model errors which pose rather conflicting constraints on the control system in terms of bandwidth, precision, mechanical vibrations, and shocks … etc.

Solutions to the problem have been actively studied in recent years in the literature [STE 02]. Many approaches are proposed. For example, repetitive structure has been shown to be very effective for rejecting repetitive disturbances [MOO 98]. However, this method requires an increased number of memory locations. Furthermore, plant uncertainty and bad knowledge of the disturbance frequency make it difficult to design a procedure providing good tracking performance. Recently a robust repetitive control was introduced in [STE 02] to solve such problem.

H∞ loop shaping is a popular design method to form the open loop (i.e. its singular values in the frequency range of interest) by introduction of weights, in order to fulfill certain aims as disturbance rejection, reference tracking, etc. However, weighting function selection is not an easy task and the order of the final controller, which is designed by this technique, is usually high [BLU 05].

The two-degree-of-freedom structure has been also shown to be very effective for rejecting disturbances and loop shaping design. The RST configuration (where R, S and T are polynomials to determine) is a widely used method in the design of controllers using classical pole placement [PAN 04]. However, it requires solving the equation of Bezout on one hand
an increased number of memory locations on the other. The Bezout equation may be solved by expanding the various polynomials as powers of the indeterminate variable and equate coefficients of like powers. This leads to a set of linear equations in the coefficients of the unknown polynomials, which are known as the Sylvester equations [DEL 93].

In [LEC 01a], [LEC 01b] a Virtual Reference Feedback Tuning (VRFT) method is used for the shaping of the sensitivity function. The VRFT is a data based method in the design of feedback controllers for a linear plant whose transfer function is unknown. It is a solution of the one degree of freedom model reference control problem. The design is based on a set of I/O data. The choice of the filter and a weighting factor used in this approach are not determined analytically but chosen on the basis of the operator's experience.

Disturbance estimator has also been known to be very effective to compensate disturbances. However, it is not very efficient when the disturbance frequency is poorly known or varying with the rotational speed. This paper proposes a new methodology based on multi-models of the disturbances where damped sine wave models are introduced. We consider the problem of rejecting sinusoidal disturbances whose magnitude and phase are unknown. The frequency is time-varying around a specific value. The proposed method combines setpoint tracking with sensitivity function loop shaping. The sensitivity function provides important information on the disturbances rejection and it constitutes a good indicator for the robustness of the controller.

1. Track following problem

A disc drive mechanism has many control loops, most of which are rather slow and of less signifiance. Track following problem is to control the position of optical spot so that it follows the desired track of the optical disc media. However, this goal is usually deviated due to the internal and external disturbances. The most important disturbances present in optical disc drive are rotation eccentricity, track irregularities, mechanical vibrations and shocks.

The position of the pick-up is controlled by two cooperative actuators; a fine actuator and a coarse actuator, which are briefly depicted in figure 1. As shown in [SHI 02], we will consider only the fine actuator because it is corrupted by the disturbance whose frequency is proportional to the disc rotation speed.

The CD-ROM drive (for LG 52X) can be represented by the following model [SHI 02]:

\[ P(s) = \frac{1.022775 \times 10^8}{s^2 + 64.73s + 166800} \tag{1} \]

This transfer function describes a voice coil motor actuator from voltage input to position output. It takes into consideration the sensor gain which converts the position displacement into voltage. The optical disc drive measures the position of the pick-up by a relative position error between the desired track and the actual position of the optical spot. The spindle motor frequency is assumed equal to 63.5 Hz.

![Figure 1. Optical disc drive scheme](https://via.placeholder.com/150)

2. The output sensitivity function

The transfer function between the disturbance \( P \) and the output \( y \) is called the output sensitivity function. It is an indicator for assessing both disturbances rejection performances and robustness [SUN 88]. In particular the inverse of its modulus is equal to the modulus margin [LAN 94].

The smaller \( |S(j\omega)| \) is, with \( \omega \in \mathbb{R} \), the more the disturbances are attenuated at the angular frequency \( \omega \). \( |S| \) is small if the magnitude of the loop gain is large. Hence, for disturbance attenuation it is necessary to shape the loop gain so it is large over those frequencies where disturbance attenuation is needed.

Making the loop gain large over a large frequency band easily results in error signals \( e \) and resulting plant inputs \( u \) that are larger than the plant can absorb. Therefore, the loop gain can only be made large over a limited frequency band. This is usually a low-pass band, that is, a band that ranges from frequency zero up to a maximal angular frequency \( W_b \) (bandwidth of the feedback loop). Effective disturbance attenuation is only achieved up to the angular frequency \( W_b \).

The larger the “capacity” of the plant is, the greater the inputs the plant can handle before it saturates are. For plants whose transfer functions have zeros with nonnegative real parts, the maximally achievable bandwidth is limited by the location of the right-half plane zero closest to the origin [FRE 85]. Figure 2 shows a typical shape of the magnitude of the sensitivity function and the complementary sensitivity function \( T \).

The complementary sensitivity function \( T(s) \)
derives its name from the equality:

\[ S(s) + T(s) = 1 \]  

(2)

This illustrates one of the basic trade-offs in feedback design: good setpoint tracking and disturbance rejection \((S = 0, T = 1)\) has to be traded off against suppression of measurement noise \((S = 1, T = 0)\).

**Figure 2. Typical form of sensitivity \(S\) and complementary sensitivity \(T\)**

It is small for low frequencies and approaches the value 1 at high frequencies. Values greater than 1 and peaking are to be avoided. Peaking easily happens near the point where the curve crosses over the level 1 (the 0 dB line). The desired shape for the sensitivity function \(S\) implies a matching shape for the magnitude of the complementary sensitivity function \(T = 1 - S\).

When \(S\) is as shown in figure 2, \(T\) is close to 1 at low frequencies and decreases to 0 at high frequencies. It may be necessary to impose further requirements on the shape of the sensitivity function if the disturbances have a distinct frequency profile.

To provide total disturbances rejection at certain frequencies, the sensitivity function value is zero at the desired frequency. We have to define a template of acceptable sensitivity function which depends on the plant to control on one hand and on the control specifications on the other. In general, the shape is based on the requirements of strong disturbances attenuation at low frequencies, and eventually a total disturbance rejection in steady state with regards to a minimum acceptable modulus margin which defines an upper boundary for the modulus of the sensitivity function.

It is desirable to make \(S(s)\) as "small" as possible. Thus \(|S(s)|\) can be made small only over a finite frequency range. The bandwidth \(W_0\) can serve as a simple closed-loop performance measure. It is related to \(S(j\omega)\) by:

\[ |S(j\omega)| < \frac{1}{\sqrt{2}} \quad \forall \omega < W_0 \]  

(3)

### 3. Stability robustness

The closed-loop system remains stable under perturbations of the loop gain as long as the Nyquist plot of the perturbed loop gain does not encircle the point \(-1\). Naturally, this may be accomplished by "keeping the Nyquist plot of the nominal feedback system away from the point \(-1\)."

The classic gain margin and phase margin are well-known indicators for how closely the Nyquist plot approaches the point \(-1\). In the classical feedback system design, robustness is often specified by establishing minimum values for the gain and phase margin. Practical requirements are \(k_m > 2\) for the gain margin and \(30^\circ < \Phi_m < 60^\circ\) for the phase margin.

The gain and phase margin do not necessarily adequately characterize the robustness. For this reason [SAI 06] introduced the modulus margin. The modulus margin \(s_m\) is the radius of the smallest circle with center \(-1\) that is tangent to the Nyquist plot. The modulus margin very directly expresses how far the Nyquist plot stays away from \(-1\).

A practical specification for the modulus margin is \(s_m > 0.5\). Adequate margin of this type is not only needed for robustness, but also to achieve a satisfactory time response of the closed-loop system.

The gain margin \(k_m\) and the phase margin \(\Phi_m\) are related to the modulus margin \(s_m\) by the inequalities:

\[ k_m \geq \frac{1}{1 - s_m}, \quad \Phi_m = 2 \arcsin\left(\frac{s_m}{2}\right) \]  

(4)

This means that if \(s_m \geq 0.5\) then \(k_m \geq 2\) and \(\Phi_m \geq 28.96^\circ\) [SAI 06]. The converse is not true in general.

### 4. Extended disturbance estimator

For instance, let us assume that the state space plant model is given by the four-tuple \((A, B, C, 0)\) completely controllable and observable of order \(n\).

\[ \dot{x} = Ax + Bu \]  

\[ y = Cx + d \]  

(5)

where \(d\) is a sinusoidal disturbance acting upon the plant output.

To reduce the influence of the disturbance \(d\) on the output \(y\), the approach is to generate an estimate of this disturbance and use this estimate as a control signal. A disturbance estimator is used to generate this estimate (fig. 3).

A typical spectrum of the disturbances consists of harmonics that are multiple of the fundamental
frequency. For control design purposes, it is therefore assumed that the disturbance signal \( d \) is a sum of \( N \) sine signals with amplitude \( A_i \), pulsation \( \omega_i \), and phase \( \varphi_i \), i.e.,

\[
d = \sum_{i=1}^{N} A_i \sin(\omega_i t + \varphi_i)
\]  

(6)

For this, the disturbance is modelled as an output of an autonomous state space model with the state transition equation:

\[
\dot{x}_d = A_d x_d
\]  

(7)

and the output equation:

\[
d = C_d x_d
\]  

(8)

The matrices \( A_d \) and \( C_d \) are given as:

\[
A_d = \begin{bmatrix} A_{d_1} & \cdots & 0 \\ 0 & \cdots & A_{d_N} \end{bmatrix}
\]  

(9)

\[
C_d = \begin{bmatrix} C_{d_1} & \cdots & C_{d_N} \end{bmatrix}
\]  

(10)

The individual block entries in these block matrices follow from the state space of a damped sine wave as:

\[
A_{d_i} = \begin{bmatrix} -\xi_i \omega_i & (\xi_i - 1) \omega_i \\ (\xi_i + 1) \omega_i & -\xi_i \omega_i \end{bmatrix}
\]  

(11)

\[
C_{d_i} = [0 \ 1]
\]  

(12)

This choice of a damped sinusoidal model is guided by the fact that this model allows to introduce two complex conjugate roots:

\[
s_i = -\xi_i \omega_i \pm j\omega_i \sqrt{1 - \xi_i^2} \quad (\xi_i \text{ is the damping factor})
\]

The damping factor of these zeros has to be scaled in accordance to the desired reduction of the shape and/or the peak of the sensitivity function. The aim of the two added complex double root is to decrease the sensitivity function magnitude in the high frequency range and to shift its maximum to a lower frequency. As a result, resonant zeros will be introduced in the sensitivity function.

With the \( N \) disturbances acting upon the plant output, an overall model of the plant with the extended estimator follows as:

\[
\dot{x}_d = A_d \dot{x}_d + \ell e
\]

\[
\dot{\hat{x}} = A \hat{x} + B u + L \hat{d}
\]

\[
\dot{\hat{y}} = C \hat{x} + \hat{d}
\]

\[
\hat{d} = C_g \hat{x}_d
\]

(13)

where \( \hat{d} \) is the reconstructed disturbance, \([L \ \ell]^T\) is the matrix gain of the estimator. This estimator design can be carried out by pole placement or by designing an optimal stationary Kalman filter [BLU 05].

The new state vector consists of the system and the states of the \( N \) disturbances model:

\[
X = [x \ x_{d_1} \cdots x_{d_N}]^T
\]  

(14)

The control law is therefore given by:

\[
u = -\hat{K}X + Jr
\]  

(15)

where \( X = [x \ x_{d_1} \cdots x_{d_N}]^T \) and \( \hat{K} = [K \ G] \) is the extended feedback gain. \( G \) is a gain vector which permits to eliminate the disturbance effect and \( J \) is a gain which provides zero steady state error.

\[
J = 1 - K(A)^{-1}B
\]

\[
- C(A)^{-1}B
\]  

(16)

So as to reject the sinusoidal disturbance, the sensitivity function has to be forced to zero. In this case, from equations (5-7), we can write

\[
\left\{ - C(sI - A + BK)^{-1} B G X_{d_i} + C_d X_{d_i} \right\}|_{t=\infty} = 0
\]  

(17)

The disturbance state can be determined using equations (7-8) and (11-12):

\[
X_{d_i} = \begin{bmatrix} \gamma_1 (s + \xi_i \omega_i) + (\xi_i - 1) \omega_i \gamma_2 \\ \gamma_2 (s + \xi_i \omega_i) + (\xi_i + 1) \omega_i \gamma_1 \end{bmatrix}
\]  

(18)

where \( \gamma_1 \) and \( \gamma_2 \) are disturbance initial conditions, i.e. \( X_{d_i}(0) = [\gamma_1 \ \gamma_2]^T \). Fixing:

\[
-C(sI - A + BK)^{-1} B = \alpha \xi_1 \omega_1 + j\beta \xi_1 \omega_1
\]  

(19)

Then equation (17) becomes:
\[
G_k = -\frac{1 + \xi_j}{1 - \xi_j - \alpha^2(\xi_j, \omega_{0j}) + \beta^2(\xi_j, \omega_{0j})} \\
G_{\omega} = \frac{\alpha(\xi_j, \omega_{0j})}{\alpha^2(\xi_j, \omega_{0j}) + \beta^2(\xi_j, \omega_{0j})}
\]  
(20)

Note that the components of the vector gain \(G\) are independent of the initial conditions of the state vector \(X_d\). This result is foreseeable because these coefficients are calculated to compensate the disturbance dynamics. It is the estimator which is assigned to adapt to the shape of the disturbance.

5. Definition of a template

We have to determine a template of acceptable output sensitivity function. This template depends on the plant to control and on the control specifications, but we propose a general shape method which ensures good robustness and good disturbance rejection. It is based on the following requirements:

- Disturbance reduction at low frequencies and total rejection in steady state.
- For specified frequencies disturbance rejection, it is necessary to shape the sensitivity so that it is large over those frequencies.
- Maximum for peak value of the magnitude of the sensitivity function \(S(j\omega)\) (which defines a minimum acceptable modulus gain).

Figure 4 shows an example of a desired sensitivity function shape. \(s_1\) corresponds to the frequency weighting function used in \(H_\infty\) and \(s_2\) the maximum peak value of the sensitivity function.

6. Loop shaping procedure

The design of a robust controller by shaping the sensitivity function can be obtained using the following procedure:

- Choose the dynamic of the state feedback and the estimator following the desired dynamics,
- Design the regulator and check the shape of the sensitivity function. Generally, make a judicious choice of the poles of the state feedback and those of the estimator for a compromise robustness / performance allows having a satisfactory modulus margin. The obtained sensitivity function will be considered as a reference concerning the estimation of the influence of the disturbances on the plant output,
- If the values of the disturbance attenuation and the bandwidth are not acceptable, introduce a damped sine wave model in the estimator. This model will introduce a pair of complex combined poles whose effect is to move the maximum of the sensitivity function towards high frequencies and to increase the disturbances rejection interval,
- If again, the values of the disturbance attenuation and the bandwidth are not acceptable or if the maximum of the sensitivity function is too strong, a solution consists of introducing other resonant roots generated by a second sinusoidal disturbance model. The effects of these additional zeros in the controller are in general a reduction of the sensitivity function around the resonant frequency of the zeros added and an increase of \(|S(j\omega)|\) magnitude in high frequency. If necessary, increase the value of the auxiliary poles and decrease the dominant poles.

7. Simulation results

To demonstrate the effectiveness of the proposed technique, several simulation examples are applied to the CD-ROM model described in section 2.

Figure 5 shows the shape of the sensitivity function for one disturbance model. Disturbance is supposed by pulsation 2 rd/s.

Figure 6 shows the effect of the increase of the bandwidth of disturbance rejection (theorem of Bode) by moving the pulsation of the model of the disturbance towards great frequencies. The choice of the first pulsation (of weak value) comes from the fact that we are interested in rejecting sinusoidal disturbances to the neighbourhood of this frequency as well as disturbances of the constant type. The second pulsation, superior to the first, is introduced to move the maximum of the sensitivity function towards superior frequencies. It is scaled on the spindle motor frequency.

We can introduce an additional model of pulsation \(\omega_{0j}\) in the estimator with a certain damping factor (fig. 7). This model will shape the sensitivity function.
around medium frequencies. The Price to be paid is a regulator of more and more raised order.

Figure 8 shows the effect of the damping factor on the shape of the sensitivity function. This additional ratio constitutes an additional parameter of very interesting regulation.

Table 1 shows that the damping factor constitutes a good parameter for disturbance tuning reduction and active damping control to achieve stability robustness (\(\omega_d = 5 \text{ rd/s, } \xi_1 = 0.5 \), \(\omega_d = 60 \text{ rd/s, } \xi_2 = 0.3\)).

Figures 9 and 10 show the controller adaptability to the disturbance angular frequency variations.

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**Figure 5. Sensitivity function shape for \(\omega_d = 2 \text{ rd/s, } \xi = 0.1\)**

**Figure 6. Sensitivity function shape for \(\omega_d = 20 \text{ rd/s, } \xi_1 = 0.1 \), \(\omega_d = 398.9823 \text{ rd/s, } \xi_2 = 0.2\)**

**Figure 7. Sensitivity function shape for \(\omega_d = 5 \text{ rd/s, } \xi_1 = 0.5 \), \(\omega_d = 60 \text{ rd/s, } \xi_2 = 0.3 \), \(\omega_d = 398.9823 \text{ rd/s, } \xi_3 = 0.12\)**

**Figure 8. Influence of the damping factor for \(\omega_d = 398.9823 \text{ rd/s (----- } \xi_3 = 0.12 \text{ and } \xi_3 = 0.4\)**

**Table 1. Damping factor versus modulus gain and sensitivity function peak (\(\omega_d = 398.9823 \text{ rd/s})**
8. Conclusion

In this paper a new methodology of design of robust controllers for SISO plants is presented. It combines pole placement and sensitivity function loop shaping. The approach is progressive and allows obtaining a loop shaping following wished frequency specifications. Thus, we showed that the fact of introducing a damping factor allows better shape of the sensitivity function. The results obtained in simulation show that the strategy provides a robust performance when disturbances and parameter uncertainties are present. Such a strategy has been derived using multi-models disturbance estimator and extended state feedback.

REFERENCES


