

Quantization Issues of Wavelet Transform Implementation and optimization with Genetic Algorithm

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Abstract: In this paper, we are motivated by the quantization of filter coefficients in the wavelet transform for image compression. First, filter coefficients are quantized in coefficient level which is based on fixed point representation. However, if coefficients are quantized, this change affects filter properties. Thus, a genetic algorithm based approach for filter coefficient quantization preserving filters properties is proposed. The genetic algorithm run twice for the quantization of high pass and low pass filter coefficients. To overcome this problem, an extension of the second approach considering the preservation of filter bank properties is proposed. It has been found that the better peak to signal noise (PSNR) is given by the filter bank quantization level. Furthermore, it gives the lowest number of non-zero bit.

Key words: Data Compression, Discrete Wavelet Transform, Data coding, Multiplierless, Genetic Algorithm, Filter Bank Implementation, Biorthogonal 9/7.

INTRODUCTION

During the last few years, many fascinating and challenging strategies of filter coefficients quantization in the optimization of Wavelet transform have been developed in order to ensure minimal area occupation of DWT implementation and minimizing implementation errors. Thus, because of the limited precision of arithmetic computers, DWT based encoders cannot achieve sincere lossless compression. So, in order to reduce implementation errors, we can use a real valued transform, in which a very large number of precision bit is devoted [8].

Lossless compression can be more efficient with the use of integer wavelet transforms [4], [5], [6] which principle features in the representation of transform coefficients by finite precision number. Thus we can ensure high performances and minimum area occupation with quantized filter coefficients.

The amount of occupied hardware can be estimated in terms of total number of non-zero bits in the representation of a coefficient. We denote this latter by N . In previous work, this subject has been treated as an optimization problem. In [7], authors search on which filter coefficients the frequency response is more sensitive in order to decide the total N terms to allocate to coefficients. However, this method has a great

implementation complexity and don't minimize material cost sufficiently. Another approach is based on approximating filter coefficients by rational ones to improve their implementation in a fixed point format [1]. In other work [2] [3], the author acts on the filter structure to give the best quantization of the filter coefficient. As a result of this work, N reaches 32 for filter bank implementation of DWT. However, N can be more optimized using a probabilistic approach based on Genetic Algorithm. In this paper we focused on the quantization of biorthogonal 9/7 DWT filter coefficients (Table I).

Table 1: 9/7 Biorthogonal filter coefficients

n	g_n	n	h_n
0	0.03782845550726	0	-0.064538826287
1	-0.02384946501956	1	-0.04068941760916
2	-0.11062440441844	2	0.41809227322162
3	0.37740285561283	3	0.78848561640558
4	0.85269867900889	4	0.41809227322162
5	0.37740285561283	5	-0.04068941760916
6	-0.11062440441844	6	-0.064538826287
7	-0.02384946501956		
8	0.03782845550726		

We start with a classical quantization method which consists in the quantization of coefficients separately. Then, we propose a Genetic Algorithm based quantization for preserving firstly filters and secondly

filter bank properties. The paper is organized as follows : In section 2 we present a review on discrete wavelet Transform, in section 3 we introduce the different levels of filter coefficient quantization. Performance measurements and discussion are developed in section 4, before concluding in section 5.

1. Discrete Wavelet Transform (DWT)

Mallat’s representation of DWT consists on tow channel perfect reconstruction filter bank as represented in figure (1) where scaling coefficients $a_{j+1;k}$ are passed through the tow channel filter bank, the filtering and downsampling operations yield the scaling and wavelet coefficients at level j at output of the analysis stage. Thus:

$$a_{j,k} = \sum_l h(l - 2k) a_{j+1,l} \tag{1}$$

$$b_{j,k} = \sum_l g(l - 2k) a_{j+1,l} \tag{2}$$

In the synthesis stage, scaling and wavelet coefficients $a_{j;k}$ and $b_{j;k}$ are treated inversely by upsampling and filtering with lowpass \tilde{G} and highpass \tilde{H} filters to perform reconstruction. This stage is also called Inverse Discrete Wavelet Transform (IDWT).

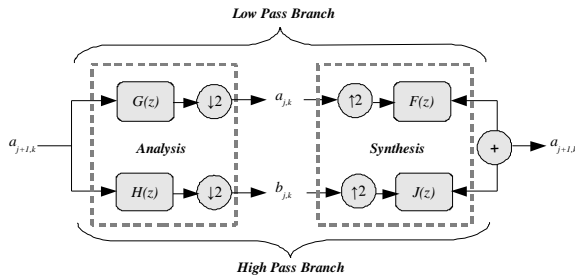


Figure 1: Conventional implementation of DWT

Original and reconstructed signal are generally different, unless the two filters H and G satisfy some relationships. The perfect reconstruction conditions consists in ensuring no distortion and no aliasing of the reconstructed data. They are given respectively [3] by :

$$F(z)G(z) + J(z)H(z) = 2z^{-d} \tag{3}$$

$$F(z)G(-z) + H(-z)J(z) = 0 \tag{4}$$

Where l is an integer delay between original signal and reconstructed signal. In biorthogonal filter bank, there exist a relationship between analysis and synthesis filters given by equations (5) and (6).

$$H(z) = F(-z) \tag{5}$$

$$J(z) = -G(-z) \tag{6}$$

Consequently, the no-aliasing perfect reconstruction condition is satisfied and the no distortion condition becomes (Equation 7):

$$G(z)F(-z) - G(-z)F(z) = 2z^{-d} \tag{7}$$

Thus, the no-distortion condition for perfect reconstruction is only related to the low filters $G(z)$ and $F(z)$.

2. Filter coefficients quantization

Filter coefficients quantization consists in a fixed point representation of filter coefficients in order to reduce the area's occupation on target and to accelerate the processing. There is usually a compromise between precision and hardware cost. We can mention three levels of filter coefficient quantization for wavelet transform: Coefficient level (level 1), Filter level (level 2) and Filter bank level (level 3).

2.1. Filter coefficients quantization: Coefficient level

In the coefficient level, each coefficient is quantized separately. With a fixed number of precision bit, we try to search the nearest fixed point number to the considered coefficient. This level is based on classical methods of fixed point representation as sum or difference of power of two. The quantization method of low pass $H(z)$ and high pass $G(z)$ filter coefficient is given in Table 2 for 7 precision bits.

Table 2: Quantized filter coefficients in coefficient level

A coefficient level quantization changes filters

	g_n		h_n
0.0390625	0.0000101	-0.0625	0.0001
-0.0234375	0.0000101	-0.0390625	0.0000101
-0.109375	0.001001	0.421875	0.011011
0.375	0.011	0.7890625	0.1100101
0.8515625	0.1101101		

frequency response. So, this leads to performance degradation. However, the more is the number of precision bit translated by an expensive hardware cost, the closer is the quantized coefficient to the real one. Thus, a compromise between occupation area and performance should be taken into account.

2.2. Filter coefficients quantization: Filter level

As a solution to the expense of hardware cost of the previous approach, we go up a higher level called filter level quantization. In this level, we search the best coefficient combination which preserves filters properties using a genetic algorithm (GA). The proposed quantization method is based on the use of genetic algorithms. Since, the genetic algorithm operates in a random way; it can sweep all the solution space. Thus, the chance to meet the good solution is very strong. GA manipulates chromosomes. Filter coefficients constitute a chromosome. A population is a set of chromosomes.

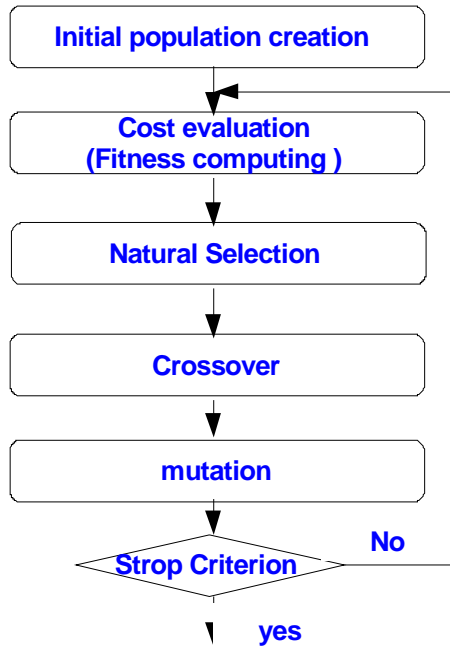


Figure 2: General scheme of genetic algorithm

General scheme of GA is presented in Figure 2. It starts with an initially chosen population. After applying the selection step using the "Wheel Selection" algorithm for this population, the remaining chromosomes are those having more chance to survive.

Table 3: Quantized filter coefficients in filter level

g _n		h _n	
0.0390625	0.0000101	-0.0703125	0.0001001
-0.0234375	0.0000101	-0.046875	0.0000101
-0.109375	0.001001	0.421875	0.100101
0.3828125	0.0110001	0.7734375	0.1100011
0.875	0.11		

These chromosomes will sudden the crossover operation. Thus, the mutation stage will be applied on the new generation resulted from the cross-over stage. After that, each chromosome will be evaluated using a cost function. This latter represents the criteria that must be optimized. It is defined in equation 8.

$$f_{cost} = \min \left(\frac{1}{2} * (s - t)^2 \right) \quad (8)$$

Where s is the frequency response of quantized filter and t is the desired response implemented with the original floating point coefficients. If this stop criteria is reached, then the run is stopped, else previous steps will be iteratively performed on resulting population until the stop criteria is satisfied. Results given by these simulations are illustrated in Table 3. In this strategy approach, the GA project runs twice for the low and the high pass filter coefficient quantization, so, the simulation time is doubled. This problem will be overcome in the following section.

2.3. Filter coefficient quantization: filter bank level

Quantized filter is part of a filter bank, this change impacts perfect reconstruction of a filter bank. The Filter bank level quantization is an extension of the second one. In this stage, we search the different coefficients that don't change the frequency response of the filter bank. Since, the no-distortion condition for perfect reconstruction is only related to the low filters G(z) and F(z). The objective function can be expressed by the same way as equation 8, for which we correspond s to the frequency response of G(z) multiplied by F(z) using the new quantized coefficients for every generation of GA. Results are presented in Table 4. In this way, optimization is done once and the simulation time is reduced.

Table 4: Quantized filter coefficient in Filter bank level

g _n		h _n	
0.0390625	0.0000101	-0.0625	0.0001
-0.03125	0.00001	-0.03125	0.00001
-0.109375	0.001001	0.421875	0.100101
0.359375	0.010111	0.78125	0.11001
0.875	0.11		

3. Performance measurements and discussion

For the sake of performance with respect to image compression applications, it is interesting to implement the three quantization levels described previously using the same test images and the same micro-processor environment. This has been achieved with the implementation of the previously-studied filter coefficients quantization levels: (i) the coefficient level, (ii) the filter level based on genetic algorithm, (iii) the filter bank level and finally the unquantized coefficients. As performance metric, we took PSNR. Results are presented in Table 5.

Table 5: Performance measurement using quantized filter coefficients

Images	Unq. Coeff.	Quantized Coefficients (7bits)			
		level 1	level 2	level 3	
wbarb	512*512	40.75	37.59	39.052	41.12
	256*256	37.71	35.73	36.89	38.09
	128*128	34.72	33.44	34.47	35.11
	64*64	31.59	30.68	31.75	31.97
woman2	32*32	28.65	27.90	29.03	29.01
	512*512	37.11	34.16	35.61	37.48
	256*256	34.08	32.22	33.43	34.46
	128*128	31.03	29.82	30.86	31.41
lena	64*64	28.01	27.15	28.18	28.4
	32*32	24.85	24.14	25.26	25.22
	512*512	38.03	34.82	36.24	38.41
	256*256	35.00	32.94	34.17	35.39
lena	128*128	31.96	30.69	31.73	32.36
	64*64	29.00	28.09	29.10	29.40
lena	32*32	25.89	25.16	26.25	26.30
	No zero bit number (N)	∞	24	22	19

Referring to Table 5, one can notice the improvement of the PSNR from one level to another. Indeed, coefficient level quantization changes filters coefficients and thus changes filters frequency response while filter bank level preserves a perfect reconstruction. Furthermore, since the image energy is mostly concentrated at low frequencies, the resemblance between low pass quantized and unquantized branch is measured at DC ($w=0$), in the pass band (See Figure 3).

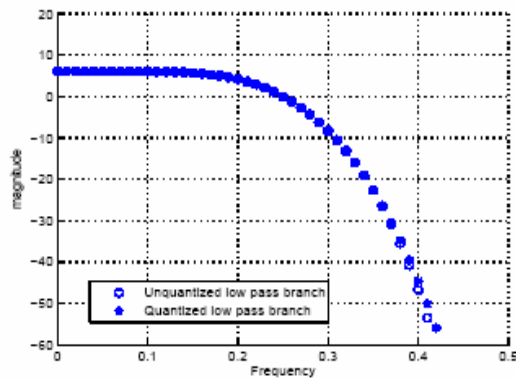


Figure 3: Resemblance between quantized and unquantized low pass branch in the pass band

Table 5, also shows the reduction of the total number of non zero bits in low and high pass filters. This value decreases from level to level leading to a decrease in hardware cost.

4. Conclusion

In this paper focused on filter coefficients quantization for wavelet transform. We first implemented a classical quantization method based on the fixed point representation of each coefficient separately. This change affects filter properties. That's why we present a filter level quantization based on genetic algorithm. We do simulations twice for the quantization of high pass and low pass filter. To overcome this problem, we proposed a filter bank level quantization to reserve perfect reconstruction conditions. Thus, the best PSNR and the lowest hardware cost are guaranteed by this last quantization level.

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