On the Use of Wavelet-Based Moment Method for Analysis of Two Dimensional Electromagnetic Scattering, Applied for Circular and Square Contour Antennas

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Abstract: The aim of this work is to introduce the application of wavelet in electromagnetic scattering, and making improvement in Moment Method development. The conventional Moment Method basis and testing functions are used to digitalize the integral equations resulting in dense matrix impedance. By using the wavelets expansions, wavelet as basis functions and testing functions, a sparse matrix is generated from the previous Moment Method dense impedance matrix. Results are compared to the previous work done and published, [Su C. And T.K. Sarkar 1998]. Excellent results are obtained for the two types of shapes circular and square contour antennas.

Key words: Moment Method, Wavelets, Sparse Matrix, Basis and Testing Functions, Integral equation.

INTRODUCTION

Numerical solutions for electromagnetic scattering problems are being increased, Moment Method is one of the most powerful numerical techniques for solving electromagnetic problems, however this method has always suffered from memory and computation time, due to digitalized integral equations resulting for very dense impedance matrix. For large antennas and great number of iteration the computational cost is unacceptable. The wavelet technique may be applied directly, in which wavelet are used as basis and testing functions [Wang Gaofeng, And Bing-Zhong Wang 1997 ] and [Zounoubi M.R, A.A. Kishk, 2005 ], or indirectly [Sokolik Dmitry, And Yair Shifman, 2004 ] and [Baharav Z. And Y.Leviatan 1998 ], which lead to sparse matrix, in either cases the computational cost is reduced drastically. Here in this contribution the wavelets are used directly using Haar wavelets type. For heavy systems and especially when using indirectly wavelets this leads to a matrix compression as done by [Pan George W. 1998 ].

The theoretical contents is expressed in the next section, which consists of three subsections, The first subsection is dedicated to integral equation and problem description, the second subsection is a bout the Moment Method formulation, the last subsection is for orthogonal wavelet expansion.

1. Formulation

1.1 Integral equation

On a two dimensional contour antennas, and for TM polarization (Hz = 0) and E = Ez(x, y). The boundary condition on the conductor surface is:

\[ Ez = E_{\text{z}} + E_{\text{s}} = 0 \]  

(1)

Where the incident field is:

\[ E_{\text{z}} = e^{jk(x \cos(\phi) + y \sin(\phi))} \]  

(2)

Using the boundary condition (1), the scattered field may be written as an integral of the induced current and the 2D Green’s function, yielding equation (3).
Where $C$ is the contour and $H_{2}^{(2)}(k|p-r|)$ is Hankel function of the second kind zero order. The two shapes studied here are presented in figure 1.

Equation (3) can be expressed as:

$$\frac{k\eta}{4} \int_{0}^{2\pi} J_{z}(r) H_{2}^{(2)}(k|p-r|) r' \theta \, dr' \, d\theta = E_{z}(r)$$  \hspace{1cm} (4)$$

Where $R=|r-r'|=\sqrt{(x-x')^2+(y-y')^2}$

For circular contour:

$$x = \frac{a}{2} \cos(\theta) \text{ and } y = \frac{a}{2} \sin(\theta).$$

For square contour:

$$\begin{cases} \frac{-\pi}{4} < \theta < \frac{\pi}{4} & \text{for } x = \frac{a}{2} \\
\frac{\pi}{4} < \theta < \frac{3\pi}{4} & \text{for } y = \frac{a}{2} \tan(\theta) \end{cases}$$

$$\begin{cases} \frac{-\pi}{4} < \theta < \frac{\pi}{4} & \text{for } x = \frac{a}{2} \cot(\theta) \\
\frac{\pi}{4} < \theta < \frac{3\pi}{4} & \text{for } y = \frac{a}{2} \end{cases}$$

1.2 Moment Method Formulation

The induced current can be found when using the Moment Method [ Balanis, C.A. 1997 ], and expand this current in a series of $N$ standard pulse basis functions, given by :

$$J_{z} = \sum_{n} I_{n} F_{n}$$  \hspace{1cm} (5)$$

Where the $F_{n}$ are the basis functions and the $I_{n}$ are the unknown constants, And using $M$ similar pulse testing functions ($g_{m}$), The inner product can be written as :

$$\sum_{n=1}^{N} \left\langle g_{m}, H_{2}^{(2)}(k|p-r|) r'(\theta) \theta \right\rangle I_{n} = \left\langle g_{m}, E_{z}^{inc}(r) \right\rangle$$  \hspace{1cm} (6)$$

Finally equation (6) can be written as a matrix form:

$$[Z_{m,n}] \cdot [I_{n}] = [U_{m}]$$  \hspace{1cm} (7)$$

Where $[Z_{m,n}]$ is the impedance matrix , and $[I_{n}]$ are the unknown constants and $[U_{m}]$ is the voltage matrix [ Lashab M. And F.Benabdellaziz 2006 ].

1.3 Wavelet expansions

The basis and testing functions are presented as a superposition of wavelets at several scales including the scaling function. A Galerkin method is then applied, where the set of basis functions used to present the current function, are used as weighting functions. The wavelets used here are Haar basis an orthogonal type. Its study is useful from theoretical point of view, because it offers an intuitive understanding of many multi-resolution properties. Furthermore, due to its simplicity Haar wavelets are widely employed. The scaling function is $\phi(x)$ , and the mother wavelet function is $\psi(x)$ , these are defined as:

$$\phi(x) = \begin{cases} 1, & \text{for } 0 \leq x \leq 1 \\
0, & \text{Otherwise} \end{cases}$$  \hspace{1cm} (8)$$

And

$$\psi(x) = \begin{cases} 1, & \text{for } 0 \leq x \leq 1/2 \\
-1, & \text{for } 1/2 \leq x \leq 1 \\
0, & \text{Otherwise} \end{cases}$$  \hspace{1cm} (9)$$

Also the scaling and the mother wavelets functions are defined by:

$$\phi_{jn}(x)=2^{j/2}\phi(2^{j}x-n)$$  \hspace{1cm} (10)$$

$$\psi_{m}(x)=2^{m/2}\psi(2^{m}x-n)$$  \hspace{1cm} (11)$$

Where ( $m$ or $j$ ) are the resolution level and ( $n$ ) is the translation factor. The vector space $V^{j}$ linear span of $\phi_{jn}$ for $j=0,1,...$And $n=0,1,...2^{j}-1$. The vector space $W^{j}$ linear span of $\psi_{m}$ for $j=0,1,...$And $n=0,1,...2^{j}-1$. The property of $W^{j} \subseteq V^{j+1}$ holds the relation between the vector spaces of different
functions by:
\[ V^{j+1} = V^j \oplus W^j \]  

(12)

The above equation states that the subspace \( W^j \) is the orthogonal complement of \( V^j \) in a larger subspace \( V^{j+1} \), which means for a given function \( f \in \mathbb{R}^N \) with \( N \) samples or \( N \) Dimensional vector, the projection of this function into the orthogonal basis is as fellow:

\[ V^k = V^0 \oplus W^0 \oplus W^1 \ldots \oplus W^{k-1} \]

For \( k = 0,1 \ldots N-1 \)

(13)

And can be expressed in an inner product as:

\[ f = \left< f, W_0 \right> + \left< f, W_1 \right> + \ldots + \left< f, W_{N-1} \right> \]

(14)

The wavelets are applied directly upon the integral equation. The density of current will be represented as a linear combination of the set wavelets functions and scaling functions as fellow:

\[ \int_{-\infty}^{\infty} \psi(x, y) \, dx = 0 \quad \forall n = 0, 1 \ldots (N-1) \]

(17)

The fact that the wavelets are orthogonal and the presence of vanishing moment, this is enabling sparse matrix production. When applying equation (6) into (12) we obtain the set of matrix equation as fellow:

(18)

Where:

(19)

(20)

(21)

(22)

Since Galerkin Method employs the same testing functions and basis function, in the same manner is the equation for incident field is expressed. After then the unknown constants are determined, and the current density can be found using equation (15), also the RCS which is directly related this latter, has been calculated by the expression (23).
\[ \sigma(\phi) = \frac{k \eta^2}{4} \int_{\theta=0}^{\theta=2\pi} |J_z(r) e^{ikr} |^2 \, d\theta \] (23)

With the assumption of:
\[ |E_{\text{incident}}|^2 = 1 \]

2. Numerical Results

A computer program has been coded in Matlab language for the technique described above, the wavelet employed is constructed from Haar orthogonal wavelet with vanishing moment N=6, the lowest resolution level is chosen \( 2^j = 2^6 = 128 \), since 128 wavelets are involved, a system of matrix ( of 128 x128 elements ) is generated. The density of current figure 4 and figure 6, and the Radar Cross Section (Bistatic RCS), figure 5 and figure 7, are presented with comparison of traditional Moment Method and Wavelet –Based MoM. Two cases of 2D contour antennas studied are circular and square shape.

Results are compared with previous work published [ Lashab M. and F.Benabdelaziz 2006 ] and [ Su C. And T.K.Sarkar 1998 ], the results are successful for diameter of size of (1.5\( \lambda \)), And a wave plane (TM case) of incident Angle \( \phi_{\text{inc}} = 180^\circ \). The results of the two methods are very close, a dimensional comparison between (figure 4, and figure 6), and (figure 5, and figure 7) show a good relation.

In this paper the use of Haar wavelet leads to a matrix sparsity of 76.2\%, for a threshold of 0.1\%, which means the moment matrices were rendered sparse by thresholding to zero all matrix elements whose magnitude was less than 0.1% of all the maximum magnitude of all matrix entries. Although [ Zahan Daniel, And Kamal Sarabanj, 2000 ] has reached 85.5\% with threshold of 3\%.

3. Conclusion

The analysis of electromagnetic scattering problem for 2D contour antennas using wavelets has
been presented for two types of shapes, circular and square. The unknown current over the conducting contour has been expanded in terms of wavelets and scaling functions. After then a sparse matrix is generated, the resolution of the wavelet is set to 2^6, going for upper resolution enable results but the impedance matrix became very heavy, so a compromise have to be made. The results obtained and compared [ Su C. And T.K. Sarkar 1998], And [Lashab M. And F.Benabdelaziz 2006] are successful.

References


