A Numerical Analysis of R-EDFA for Long Haul Optical Fiber Communication System

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Abstract: This paper presents a simple and efficient design process of a remotely pumped optimized Erbium doped fiber amplifier (R-EDFA) for repeaterless long haul optical fiber communication system (OFCS) by the numerical analysis of EDFA rate equation model. The length of Erbium doped fiber (EDF) is optimized using very low remote pump power. The effect of pump power, signal power, signal wavelength and amplified spontaneous emission (ASE) on the gain and noise figure (NF) of R-EDFA is described in details using the numerical simulation of the EDFA rate equation model. Using these findings as well as considering high gain and low NF of R-EDFA as main design objectives, the designers are able to carry out the design of an optimized R-EDFA for long haul OFCS by following the proposed design process.

Key words: Erbium-doped fiber amplifier, Optical fiber communication system, Remote pump power.

INTRODUCTION

The availability of practical laser diode pump sources have made EDFAs ideal for 1550 nm long haul OFCS [Olsson, N. A. 1989]. The attractive features of EDFAs consist in their high gain, wide optical bandwidth, high-output saturation, near quantum limited noise, low insertion loss, polarization-independence and immunity to saturation-induced crosstalk [Mao, Q., Wang, J., Sun, X. and Zhang, M. 1999]. The R-EDFAs are attractive choice in repeaterless long haul OFCS where the establishment of inline power station is difficult like undersea communication. Because of the high importance of R-EDFAs in modern repeaterless long haul OFCS, a detailed analysis tool is essential in order to obtain the results of the system performance. Both analytical models [Zech, H. 1995] [Desurvire, E. 1996] and numerical models [Mao, Q., Wang, J., Sun, X. and Zhang, M. 1999] [Ko, K. Y., Demokan, M. S. and Tam, H. Y. 1994] [Zhang, X. and Mitchell, 2000] have been reported previously for EDFA. This paper focuses on the optimization of R-EDFAs for repeaterless long haul OFCS by the numerical solution of rate equation model [Giles, C. R. and Desurvire, E. 1991] of EDFA. In this regard a systematic design process with example is described.

1. R-EDFA Configuration

The basic configuration of single pass (SP) R-EDFA is depicted in figure-1. In order to focus on the optimized design procedure, a simple SP configuration is considered in this present work. Forward pumping scheme with respect to the direction of the input signal has been used to design the amplifier.

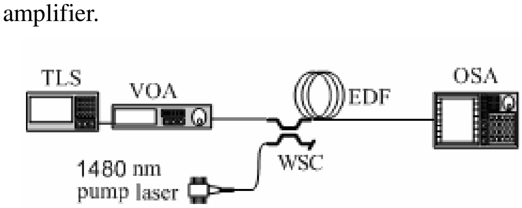

Figure 1: SP R-EDFA configuration.

In general, a 980 nm pump provides higher gain and lower noise figure than a 1480 nm pump [BECKER, P. C. 2002]. Since our proposed design is for the remote pump application and attenuation is higher for a 980 nm pump than a 1480 nm pump [Desurvire, E. 1994], a 1480 nm pump is considered. A wavelength selective coupler (WSC) is used to combine the 1480 nm pump with the test signal. A tunable laser source (TLS) is used as a source of the signal, variable optical attenuator (VOA) is used to attenuate the signal power as well as optical spectrum analyzer (OSA) is used to measure the amplified signal power and NF.

2. Model

Since the pumping at 1480 nm populates the upper amplifier level $^4I_{13/2}$ of the Er^{3+} ions directly, a two level transition between $^4I_{15/2}$ - $^4I_{13/2}$ is considered. We assume that the EDF medium is homogeneously broadened, ASE for polarization states as well as signal, pump and ASE are propagating in the fundamental mode. The population densities N_1 and N_2 of the $^4I_{13/2}$ and $^4I_{15/2}$ can be calculated as [Desurvire, E. 1994]:

$$N_1 = \rho \frac{1+W_{21}\tau_{spon}}{1+(W_{12}+W_{21})\tau_{spon}+R\tau_{spon}} \quad (1)$$

$$N_2 = \rho \frac{R\tau+W_{12}\tau_{spon}}{1+(W_{12}+W_{21})\tau_{spon}+R\tau_{spon}} \quad (2)$$

Where W_{12} and W_{21} are the up and down stimulated transition rates respectively, R is the pumping rate, τ is the fluorescence lifetime and by definition, $\rho = N_1 + N_2$ is the Er^{3+} ion density per unit volume .Let us consider, $\sigma_{SE}(\lambda_S)$, $\sigma_{SA}(\lambda_S)$, $\sigma_{PE}(\lambda_P)$, $\sigma_{PA}(\lambda_P)$ are the emission and absorption cross sections at signal and pump frequencies V_S and V_P respectively. Γ_S and Γ_P are the overlap factor, representing the overlap between the erbium ions and the mode of the signal light field and pump light field respectively, and A is the effective cross-sectional area of the distribution of erbium ions. The value of W_{12}, W_{21} and R can be calculated as:

$$W_{12} = \frac{\sigma_{SA}(\lambda_S)\Gamma_S}{hV_S A}\left[P_S + P^+_{ASE} + P^-_{ASE}\right] \quad (3)$$

$$W_{21} = \frac{\sigma_{SE}(\lambda_S)\Gamma_S}{hV_S A}\left[P_S + P^+_{ASE} + P^-_{ASE}\right] \quad (4)$$

$$R = \frac{P_P^+\Gamma_P\sigma_P(\lambda_P)}{hV_P A} \quad (5)$$

Where h is the Plank constant, P_S is the signal power, P_P^+ is the forward pump power as well as P^+_{ASE} and P^-_{ASE} are the forward and backward spontaneous emission spectrum of SP R-EDFA. The equations describing the spatial development of P_S , P_P^+, P^+_{ASE} and P^-_{ASE} can be written as based on the Giles and Desurvire model [Giles, C. R. and Desurvire, E. 1991]:

$$\frac{dP_p^+}{dz} = P_p^+\Gamma_p\left(\sigma_{PE}(\lambda_P)N_2 - \sigma_{PA}(\lambda_P)N_1\right) - \alpha_p P_p^+ \quad (6)$$

$$\frac{dP_S}{dz} = P_S\Gamma_S\left(\sigma_{SE}(\lambda_S)N_2 - \sigma_{SA}(\lambda_S)N_1\right) - \alpha_S P_S \quad (7)$$

$$\frac{dP^+_{ASE}}{dz} = \begin{aligned}&P^+_{ASE}\Gamma_S\left(\sigma_{SE}(\lambda_S)N_2 - \sigma_{SA}(\lambda_S)N_1\right)\\ &+2\sigma_{SE}(\lambda_S)N_2\Gamma_S hV_S\Delta v - \alpha_S P^+_{ASE}\end{aligned} \quad (8)$$

$$\frac{dP^-_{ASE}}{dz} = \begin{aligned}&-P^-_{ASE}\Gamma_S\left(\sigma_{SE}(\lambda_S)N_2 - \sigma_{SA}(\lambda_S)N_1\right)\\ &+2\sigma_{SE}(\lambda_S)N_2\Gamma_S hV_S\Delta v + \alpha_S P^-_{ASE}\end{aligned} \quad (9)$$

Where z is the co-ordinate along the EDFA. The second term on the right hand side of equation (8) and (9) is the spontaneous noise power produced in the amplifier per unit length within the amplifier homogeneous bandwidth Δv for both polarization states and α_S and α_P represents the internal signal and pump loss term of the amplifier respectively. Noise figure is closely related to ASE, which is generated by through spontaneous emission and the number of spontaneous photons are given by [BECKER, P. C. 2002]:

$$\eta_{SP} = \frac{\eta N_2}{\eta N_2 - N_1} \quad (10)$$

Where η_{SP} is known as the spontaneous emission factor and $\eta = \frac{\sigma_{SE}}{\sigma_{SA}}$. The noise figure of SP R-EDFA (NF(λ_S)) at the signal wavelength λ_S can be calculated as:

$$NF(\lambda_S) = \frac{1+2\eta_{SP}[G-1]}{G} \quad (11)$$

Where G is the gain of SP R-EDFA. For high gain condition (G>20 dB) equation can be written as [Desurvire, E. 1994]:

$$NF(\lambda_S) \approx 2\eta_{SP} \quad (12)$$

3. Numerical Resolution

The first order differential equations (6) – (9) are two boundary value problem. First we have used Runge-Kutta method to obtain a set of approximate solution and then relaxation method is used to make iterative adjustment to the solution. Initially we have performed the integration from z = 0 to z = L, without considering P^-_{ASE} , i.e. P_P^+ (z = 0) = $P^+_{P\_initial}$ (initial pump power to R-EDFA), P_S (z = 0) = $P_{S\_initial}$ (initial signal power to R-EDFA) and P^+_{ASE} (z = 0) = 0 as the initial boundary value. The whole set of equation including P^-_{ASE} are then integrated from z = L to z = 0 with P^-_{ASE} (z = L) = 0. For the accuracy of these quasisolutions we repeated the same procedure using relaxation method to make iterative adjustment to the solution. The EDF parameters used to simulate the model is shown in Table – I.

λ_S =1550 nm.	$\sigma_{SA}(\lambda_S)$ = 2.910556003 X10^{-25} m^2
λ_p = 1480 nm.	$\sigma_{SE}(\lambda_S)$ = 4.118853202 X10^{-25} m^2
Γ_S = 0.74	$\sigma_{PA}(\lambda_P)$ = 2.787671233 X10^{-25} m^2
Γ_P = 0.77	$\sigma_{PE}(\lambda_P)$ = 0.810563905 X10^{-25} m^2
A = 1.633 X 10^{-11} m^2	Δv = 3100 GHz (25 nm)
ρ = 300 ppm	τ_{Spon} = 0.0102 seconds
α_S =0.20 dB/Km	α_P = 0.24 dB/Km

TABLE 1: EDF PARAMETERS FOR SIMULATION.

4. Results and Discussion

In case of remotely pumped repeaterless long haul OFCS, location of the R-EDFA is far away from the remote pump source and for this reason the necessity of very low pump power is essential. Initially the designers need to determine the available pump power at the input of the R-EDFA. A very low remote pump power of 10 mW is considered at the input of the R-EDFA in this work. Now the length of the EDF is needed to be optimized with respect to the input pump power to achieve maximum gain and low NF. Upper (N2) and ground (N1) state population can be calculated with respect to the various EDF lengths from the numerical simulation. Figure -2 shows the population density in per cubic meter in the upper state and ground state as a function of position along a 20 m long EDF using 10 mW pump power.

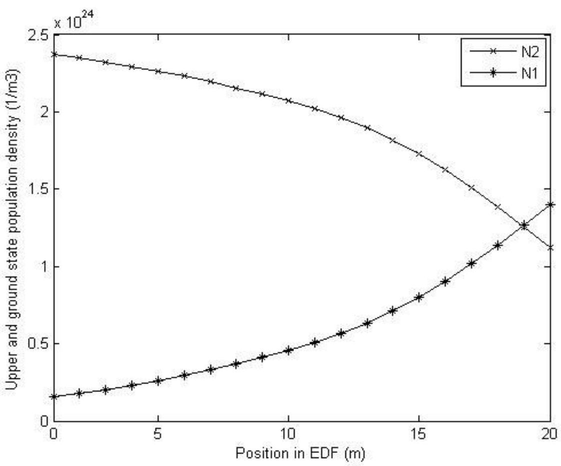

Figure 2: Population in the upper state (N2) and ground state (N1) as a function of position along a 20 m long EDF at 1550 nm using 10 mW of pump power and injected signal power of -35 dBm.

From this figure, after 19 meter length, upper state population is less than the ground state population. For this reason, if we use an EDF of length more than 19 meter then the portion of the EDF that exceeds 19 meter remains unpumped. This unpumped portion of the EDF absorbs the signal and degrades the system performance. Moreover because of the additional length of EDF, backward ASE travels over a longer distance and become much higher at the beginning of the EDF. So an EDF of length more than 19 meter causes higher backward ASE which depletes the inversion and robs gain at the expense of the signal. On the other hand, if an EDF of length less than 19 meter is used for the proposed remotely pumped SP EDFA configuration then a portion of the pump power will remain unused which can causes more population inversion and hence the increment of the gain. For these reasons an EDF length of 19 meter is chosen as an optimized length for the proposed remotely pumped SP EDFA configuration.

Figure -3 shows the signal gain as a function of EDF length at 1550 nm using 10 mW of pump power and injected signal power of -35 dBm. Referring to figure 3, signal gain increases upto the length of 19 meter and after the 19 meter it begins to reduce again which justifies the findings in figure - 2.

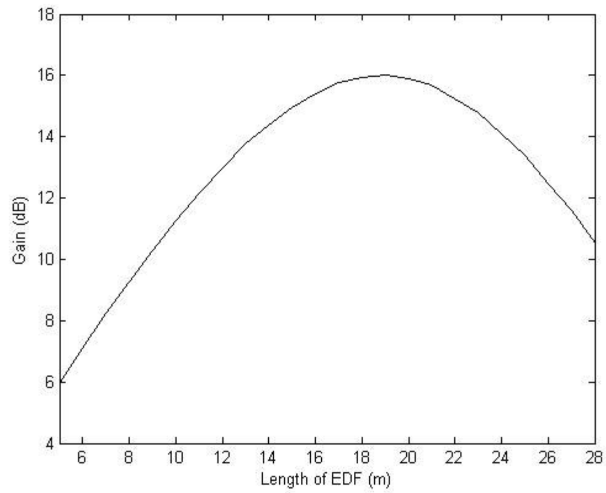

Figure 3: Signal gain as a function of EDF length at 1550 nm using 10 mW of pump power and injected signal power of -35 dBm.

Figure -4 shows the signal gain and NF in dB as a function of pump power in mW using a 19 m long EDF at 1550 nm signal wavelength. From this figure, gain values are gradually increased and NF values are gradually decreased with the increment of pump power. This is because the population inversion increases with the increment of pump power.

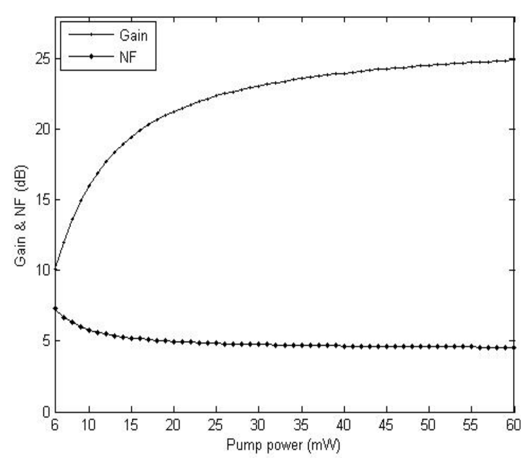

Figure 4: Signal gain and NF in dB as a function of pump power in mW using a 19 m long EDF at 1550 nm signal wavelength and injected signal power of -35 dBm.

After certain pump power, upper state population reaches almost to a constant level and for this reason after a certain pump power gain and NF values become saturated which is noticed in the figure -4. Figure -5 shows that the gain values are gradually decreased and NF values are increased dramatically with the increment of signal power.

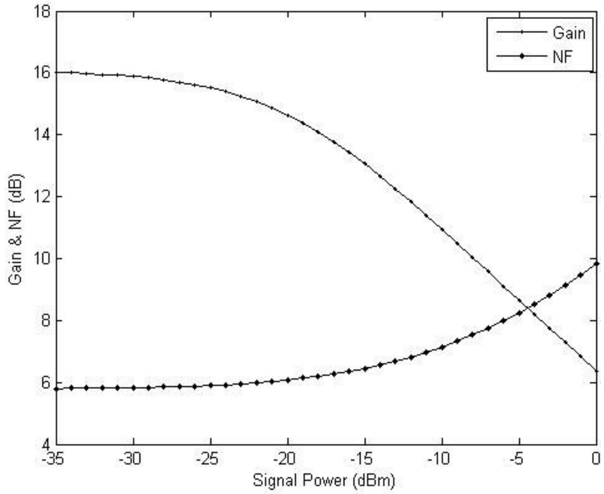

Figure 5: Signal gain and NF in dB as a function of signal power in dBm using a 19 m long EDF at 1550 nm signal wavelength and injected pump power of 10 mW**.**

At higher input signal power levels, the strong signal significantly depletes the inversion and the pump is not able to replenish it as a result the gain decreases and NF increases rapidly with signal power. Figure -6 shows the signal gain in dB as a function of signal wavelength for the pump powers indicated on the graphs and signal input power -35 dBm using a 19 m long EDF. From this figure, gain for 1530 nm is high due to its higher emission cross section. The spectral shapes of the gain change nonuniformly with the changes in pump power. In particular, as pump powers decrease, signals near 1530 nm will experience a drop in gain much more significant than that for signals near 1550 nm as shown in figure 6.

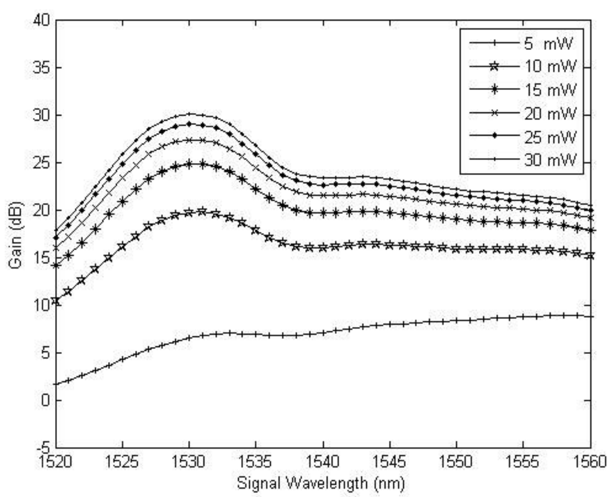

Figure 6: Signal gain in dB as a function of signal wavelength (signal power input -35 dBm), for the pump powers indicated on the graphs using a 19 m long EDF**.**

5. Conclusion

The physical bases of R-EDFA in long haul OFCS has described in details. The effects of pump power, signal power and signal wavelength on the gain and noise figure of the R-EDFA have analyzed using the numerical simulation. These numerical results will play an important role to design a practical C-band R-EDFA for the long haul OFCS from the point of view of optimal design of R-EDFA.

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