



An Efficient Electromagnetic Study of Circular Disc Microstrip Antenna with Two Parasitic Elements

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Abstract: In this paper, a rigorous analysis of three stacked circular microstrip patches is performed using a dyadic Green's functions formulation. The circular discs are embedded in a multilayered substrate containing isotropic and/or uniaxial anisotropic materials. A very efficient technique to derive the dyadic Green's functions of the stacked configuration in the vector Hankel transform domain is proposed. The complete orthogonal set of transverse magnetic (TM) and transverse electric (TE) modes of a cylindrical cavity with magnetic side walls and electric top and bottom walls is used in the approximation of the unknown current densities on the circular discs. Numerical results obtained for the resonant frequencies are found to be in good agreement with the experimental data available in the literature. **Key words:** Circular patches, Green's functions, multilayered medium, stacked configuration.

INTRODUCTION

Extensive research has been aimed at improving the bandwidth and gain of microstrip antennas. Among the various schemes that have been proposed, the stacked geometry appears to be very promising. By proper design, it is capable of providing dualfrequency characteristics [Losada 03], wider bandwidth [Khodier 03], and higher gain [Tulintseff 1991]. Theoretical investigations of two stacked microstrip patches are available in the literature, however, for stacked configurations involving three patches, any exact or approximate analysis has never been done due to the complexity of the structure.

Apart from stacked microstrip patches, in the last few years, there has been a growing interest in analyzing microstrip patches on anisotropic substrates [Fortaki 04]. Bearing in mind that many practical materials used as substrates of printed circuit and antennas exhibit a significant amount of anisotropy that can affect the performance of microstrip patches, accurate characterization and design must account for this effect.

This paper presents a rigorous full-wave analysis of three stacked circular microstrip patches fabricated on a multilayered substrate containing isotropic and/or uniaxial anisotropic materials. This paper is organized as follows. In section 1, a new efficient approach to derive the dyadic Green's functions of the problem is proposed. In section 2, To validate the proposed theory, our numerical results obtained for the resonant frequencies are compared with experimental data. Finally, conclusions are summarized in section 3.

1. Theory

The problem to be solved is illustrated in Figure 1. We have three stacked circular microstrip patches of radii a_1 , a_2 , and a_3 fabricated on a grounded multilayered substrate. The circular discs and the ground plane are assumed to be infinitesimally thin and perfectly conducting and the layers are taken to be infinite in extent. The multilayered medium is constituted of N uniaxial anisotropic substrates with the optical axis normal to the patches. Each layer of thickess $d_j = z_j - z_{j-1}$ is characterized by the free-space permeability μ_0 and a permittivity tensor of the form

$$\overline{\boldsymbol{\varepsilon}}_{j} = \boldsymbol{\varepsilon}_{0} \operatorname{diag} \left[\boldsymbol{\varepsilon}_{xj}, \, \boldsymbol{\varepsilon}_{xj}, \, \boldsymbol{\varepsilon}_{zj} \right] \tag{1}$$

where ε_0 is the free-space permittivity and diag stands for the diagonal matrix with the diagonal elements appearing between the brackets. Equation (1) can be specialized to the isotropic substrate by allowing $\varepsilon_{xj} = \varepsilon_{zj} = \varepsilon_{rj}$. The patches are embedded in the stratification at the interface planes $z = z_p$,



Figure 1. Geometrical structure of three stacked circular microstrip patches in a multilayered dielectric medium that contains isotropic and/or uniaxial anisotropic materials.

 $z = z_{P_2}$, and $z = z_{P_3}$. All fields and currents are time harmonic with the $e^{i\omega t}$ time dependence suppressed.

Let
$$\mathbf{J}^{i}(\rho,\phi) = \begin{bmatrix} J^{i}_{\rho}(\rho,\phi) & J^{i}_{\phi}(\rho,\phi) \end{bmatrix}^{T}$$

(where T implies transpose and i=1,2,3) be the surface current density on the disc of radius a_i . Also,

let
$$\mathbf{E}^{i}(\rho, \phi, z_{P_{i}}) = \left[E^{i}_{\rho}(\rho, \phi, z_{P_{i}}) \qquad E^{i}_{\phi}(\rho, \phi, z_{P_{i}}) \right]$$

be the value of the transverse electric field at the plane of the patch of radius a_i . Owing to the revolution symmetry of the multilayered medium of Figure 1 around the z-axis, when the Helmholtz equations for the longitudinal field components E_z and H_z are solved in cylindrical coordinates inside each of the layers of that medium, it turns out that the dependence of E_z and H_z on the ϕ coordinate is of type $e^{ik\phi}$ (where k is an integer), as a consequence, $\mathbf{J}^i(\rho, \phi)$ and $\mathbf{E}^i(\rho, \phi, z_p)$ can be written as

$$\mathbf{J}^{i}(\rho,\phi) = \sum_{k=-\infty}^{+\infty} e^{\mathbf{i}\,k\phi} \,\mathbf{J}^{i}_{k}(\rho)$$
(2)

$$\mathbf{E}^{i}(\rho,\phi,z_{P_{i}}) = \sum_{k=-\infty}^{+\infty} e^{i\,k\phi} \,\mathbf{E}^{i}_{k}(\rho,z_{P_{i}})$$
(3)

Following a mathematical reasoning similar to that shown in [Fortaki 04, Eqs. (2)-(22)], we obtain a relation among $\mathbf{J}^{1}(\rho, \phi)$, $\mathbf{J}^{2}(\rho, \phi)$, $\mathbf{J}^{3}(\rho, \phi)$, $\mathbf{E}^{1}(\rho, \phi, z_{P_{1}})$, $\mathbf{E}^{2}(\rho, \phi, z_{P_{2}})$, and $\mathbf{E}^{3}(\rho, \phi, z_{P_{3}})$ in the spectral domain given by

$$\mathbf{e}_{k}^{1}(k_{\rho}, z_{P_{1}}) = \overline{\mathbf{G}}^{11}(k_{\rho}) \cdot \mathbf{j}_{k}^{1}(k_{\rho}) + \overline{\mathbf{G}}^{12}(k_{\rho}) \cdot \mathbf{j}_{k}^{2}(k_{\rho}) + \overline{\mathbf{G}}^{13}(k_{\rho}) \cdot \mathbf{j}_{k}^{3}(k_{\rho}) \quad (4)$$
$$\mathbf{e}_{k}^{2}(k_{\rho}, z_{P_{2}}) = \overline{\mathbf{G}}^{21}(k_{\rho}) \cdot \mathbf{j}_{k}^{1}(k_{\rho}) + \overline{\mathbf{G}}^{22}(k_{\rho}) \cdot \mathbf{j}_{k}^{2}(k_{\rho}) + \overline{\mathbf{G}}^{23}(k_{\rho}) \cdot \mathbf{j}_{k}^{3}(k_{\rho}) \quad (5)$$
$$\mathbf{e}_{k}^{3}(k_{\rho}, z_{P_{3}}) = \overline{\mathbf{G}}^{31}(k_{\rho}) \cdot \mathbf{j}_{k}^{1}(k_{\rho}) + \overline{\mathbf{G}}^{32}(k_{\rho}) \cdot \mathbf{j}_{k}^{2}(k_{\rho})$$

+
$$\overline{\mathbf{G}}^{33}(k_{\rho})$$
. $\mathbf{j}_{k}^{3}(k_{\rho})$ (6)

where $\mathbf{j}_{k}^{i}(k_{\rho})$ and $\mathbf{e}_{k}^{i}(k_{\rho}, z_{P_{i}})$ are, respectively, the vector Hankel transforms of $\mathbf{J}_{k}^{i}(\rho)$ and $\mathbf{E}_{k}^{i}(\rho, z_{P_{i}})$, and the *nm* element of the dyadic Green's functions is given by

$$\overline{\mathbf{G}}^{nm}(k_{\rho}) = \overline{\mathbf{\Gamma}}_{< n}^{12} \cdot \left[\overline{\mathbf{g}}_{0} \cdot \overline{\mathbf{\Gamma}}_{> m}^{12} - \overline{\mathbf{\Gamma}}_{> m}^{22}\right] \cdot \left[\overline{\mathbf{g}}_{0} \cdot \overline{\mathbf{\Gamma}}^{12} - \overline{\mathbf{\Gamma}}^{22}\right]^{-1} = \overline{\mathbf{G}}^{mn}(k_{\rho})$$

$$n = 1, \dots, 3; \ m = n, \dots, 3$$
(7)

with

$$\overline{\Gamma}_{< n} = \prod_{j=P_n}^{1} \overline{\mathbf{T}}_j , \quad \overline{\Gamma}_{> m} = \prod_{j=N}^{P_m+1} \overline{\mathbf{T}}_j , \quad \overline{\Gamma} = \prod_{j=N}^{1} \overline{\mathbf{T}}_j$$
(8)

In equation (8) $\overline{\mathbf{T}}_i$ is the matrix representation of the

 j^{th} layer in the (TM,TE) representation [Fortaki 04, Eq. (9)]. Whatever the number of layers in the stacked configuration, the new explicit expression shown in equation (7) allows the computation of the dyadic Green's functions easily using simple matrix multiplications.

Now that we have the necessary Green's functions, the well-known Galerkin's procedure of the moment method can be easily applied to equations (4), (5), and (6) to obtain the complex resonant frequencies of the resonant modes of the structure shown in Figure 1.

2. Results

In this section, we apply the formulation of section 1 to the stacked geometry of Figure 2. In order to achieve tunable resonant frequency characteristic, two adjustable air gap layers are inserted in the configuration. The space-domain basis functions are chosen to be the TM and TE current modes of the three magnetic-wall cylindrical cavities with radii a_1 ,

 a_2 , and a_3 . Some spurious resonances are found in



Figure 2. Geometrical structure of the stacked configuration used in the experiment of Revankar and Kumar [Revankar 1991].

the stacked microstrip antennas but not in antennas involving a single conductor patch.

Numerical results are obtained for the parameters used in the experiment of Revankar and Kumar [Revankar 1991], i.e., $a_1 = a_2 = a_3 = 1.65 \text{ cm}$, $\varepsilon_{r_1} = 2.33$, $\varepsilon_{r_3} = 2.45$, $\varepsilon_{r_5} = 2.2$, $d_1 = 1.58$ mm, $d_2 = 4 \text{ mm}$, $d_3 = 0.762 \text{ mm}$, and $d_5 = 0.508 \text{ mm}$. The antenna is then characterized by varying the air separation d_{A} . The calculated resonant frequencies of the TM₁₁ mode are shown in table 1 and are compared with the measured values of Revankar and Kumar. The presence of the parasitic patches introduces two resonant frequencies, namely the lower (f_i) and upper (f_{μ}) frequencies. Note that the measured resonant frequencies f_l and f_u given in table 1 are obtained from the curves of the return loss. It is seen that our calculated resonant frequencies differ from the measured results by at most 6.06%. Consequently, a good agreement between theory and experiment is achieved.

Table 1. Comparison between calculated and measured resonant frequencies of the stacked configuration shown in Figure 2; for different values of the air separation d_4 .

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	Resonant frequencies (GHz)			
<i>d</i> ₄ (mm)	Measured		Our calculations	
	[Revankar 1991]			
	f_l	f_u	f_l	f_u
7.4	3.2750	3.4225	3.1682	3.5388
6.4	3.2825	3.4250	3.1677	3.5987
5.2	3.3125	3.4575	3.1691	3.6554
4.2	3.3000	3.5000	3.1823	3.7122
3.2	3.2725	3.6500	3.1851	3.7581
2.0	3.2725	3.7000	3.1890	3.8412
1.0	3.3000	3.7750	3.1984	3.9001
0.5	3.3125	3.7875	3.2201	3.9386



Figure 3. Lower and upper resonant frequencies for the stacked geometry of Figure 2 versus the air separation d_{γ} .

In table 1 we have given the lower and upper resonant frequencies for different values of the air separation d_4 . Now the influence of the air separation d_2 on the lower and upper resonant frequencies of the stacked geometry of Figure 2 is studied in Figure 3. The antenna parameters are: $a_1 = a_2 = a_3 = 2 \text{ cm}$, $\varepsilon_{r1} = 2.33$, $\varepsilon_{r3} = 2.45$, $\varepsilon_{r5} = 2.2$, $d_1 = 1.58$ mm, $d_3 = 0.762 \text{ mm}$, $d_4 = 4 \text{ mm}$, and $d_5 = 0.508 \text{ mm}$. The antenna is then characterized by varying the air gap width d_2 . It is observed that when the air gap width d_2 grows, the upper resonant frequency decreases considerably, while the lower resonant frequency is almost constant. As an example, when the air gap width is changed from 1 mm to 6 mm, the upper (lower)resonant frequency decreases from 3.244 GHz (2.661 GHz) to 3.012 GHz (2.636 GHz) for a large (small) fractional change of 7.15 % (0.94 %).

3. Conclusion

This paper has described a rigorous full-wave analysis of three stacked circular microstrip patches fabricated on a multilayered substrate containing isotropic and/or uniaxial anisotropic materials. A very efficient technique to derive the dyadic Green's functions of the stacked configuration in the (TM,TE) representation has been proposed. Numerical resonant frequencies have been shown to be in a good agreement with experimental data, so that this analysis method can be thought of as a usable tool for the design of this type of antennas. This analysis method can be applied to more general systems.

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