COMET and MUSIC Performance Comparison in the Presence of a Diffuse Source

Insaf Jaafar, Hatem Boujemâa and Mohamed Siala

SUPCOM, Route de Raouad Km 3,5 2083 El Ghazala, Ariana, Tunisia

Abstract: In this paper, the problem of diffuse source localization is considered. Conventional estimation methods are based on the assumption of point sources. However, in wireless communication scenarios, local scattering in the source vicinity causes angular spreading which degrades the performance of conventional localization algorithms like MUSIC and ESPRIT. In this paper, we compare the performance of MUSIC and COMET (COvariance Matching EsTimator) in terms of Root Mean Square Error (RMSE) of the direction of arrival (DOA) and the angular spread estimates.

Keywords: Angular spread, COMET algorithm, DOA, Local scattering, MUSIC algorithm, RMSE.

1. Introduction

The use of antenna arrays at the base station in wireless communication systems has gained much interest in the recent decades. With the increasing demand for wireless communications, much attention has recently been paid to the use of antenna arrays in order to exploit the spatial dimension leading to an increase in the capacity (L. C. Godara, 1997, A. J. Paulraj & al. 1997). With a base station equipped with multiple antennas, beamforming can be performed for DOA estimation enabling source geolocation. In the field of array signal processing, a class of DOA estimation method has been developed based upon the eigenstructure of the spatial covariance matrix. For example, conventional subspace algorithms for 2-D DOA estimation methods such as MUltiple Signal Classification (MUSIC) (R. O. Schmidt, 1981) and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) (R. Roy & al. 1989) have been developed. These algorithms are based on signal and noise subspace decomposition of the spatial covariance matrix. In realistic mobile environments, local scattering in the mobile vicinity causes angular spreading to arrive via different paths and at different angles. So, point source localization methods lead to erroneous results when directly applied to distributed source localization (T. P. Jäntti, 1992). In (R. Moses & al. 1995), the bias and RMSE of MUSIC and ESPRIT estimates have been studied in the case of rapidly time varying channels and in (D. Astély & al. 1999, I. Jaafar & al. 2004) for slowly time-varying channels.

In order to improve the quality of DOA estimates, the Maximum Likelihood (ML) technique to estimate the parameters of angular spread distribution is described (T. Trump & al., 1996) and the corresponding Cramer-Rao Bound (CRB) is provided. Since the complexity of the ML algorithm is high, suboptimal approaches like COMET (COvariance Matching EsTimator) (T. Trump & al., 1996) have been proposed. In this paper, we propose to compare the performance of MUSIC to that of WLS (Weighting Least Square) and LS (Least Square) algorithms which belong to the class of COMET approaches. The obtained results are also compared to the CRB and LS theoretical analysis.

The paper is organized as follows. Section 2, presents the scattering model. A brief recall of the MUSIC algorithm is given in section 3. In section 4 the COMET algorithm is described. In section 5 the Cramer Rao Bound (CRB) of the DOA and the angular spread estimated are derived. Numerical and simulation results are given in section 6. Finally, section 7 draws some conclusions.

2. Scattering model

Assume a large number of wavefronts impinging on the antenna array, all originating from independent reflections in the source vicinity. In the presence of a
single source, the received signal by an array of \( M \) sensors is given by
\[
x(t) = s(t) + \sum_{k=1}^{K} \alpha_k a(\theta + \hat{\theta}_k) + n(t),
\]
where \( x(t) = [x_1(t), \ldots, x_M(t)]^T \), \( T \) is the transpose operator, \( n(t) \) is a Gaussian noise with zero mean and covariance matrix \( E = \{n(t)/n(t)\} = \sigma^2 I_M \), where \( H \) is the hermitian transpose operator, \( \alpha_k \) is the complex ray gain which is assumed to be independent from snapshot to snapshot as well as from ray to ray, \( s(t) \) is the signal transmitted by the source, \( L \) is the total number of local scatterers associated with the source, \( \theta \) is the nominal direction of the source, \( \hat{\theta}_k \) is a small deviation from the nominal direction, and \( a(\theta) \) is the array manifold for a Uniform Linear Antennas (ULA), we have
\[
a(\theta) = \begin{bmatrix} 1, e^{-2\pi d_f/(M-1) \sin\theta} & \ldots & e^{-2\pi d_f/M \sin\theta} \end{bmatrix}^T,
\]
where \( d \) is the distance between two successive sensors, \( f_c \) is the carrier frequency and \( c \) is the light speed.

In the absence of local scattering, the spatial covariance matrix \( \mathbf{R} = [x(t)x^H(t)] \) is given by
\[
\mathbf{R}_0 = a(\theta) S a^H(\theta) + \sigma^2 I_M,
\]
where \( S = E[|s(t)|^2] \).

In the presence of a Gaussian diffuse source, the \( (k,n) \)-th element of the spatial covariance matrix is given by (T. Trump & al., 1996)
\[
\mathbf{R}_k(k,n) = SE \sum_{\ell} \sum_{\rho} \alpha_\ell \alpha_\rho^H
\]
\[
\cdot a_k(\theta + \hat{\theta}) a_n(\theta + \hat{\theta}) + \sigma^2 \delta_{k,n},
\]
where \( \delta_{k,n} \) is the Kronecker symbol.

Since the complex gains are assumed to be independent, we deduce
\[
\mathbf{R}_k(k,n) = \frac{S}{\sqrt{2\pi \sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{\theta^2}{2\sigma^2}}
\]
\[
\cdot a_k(\theta + \hat{\theta}) a_n^H(\theta + \hat{\theta}) d\theta + \sigma^2 \delta_{k,n},
\]
where \( \sigma_\theta \) the standard deviation of the source DOA. Using a first order Taylor expansion, the \( (k,n) \)-th element of \( \mathbf{R}_k \) is given by
\[
\mathbf{R}_k(k,n) = S \exp[-2\pi \Delta(k-n) \sigma_\theta \cos\theta]^2
\]
\[
\cdot \exp[-2\pi j\Delta \sin\theta (k-n)] + \sigma^2 \delta_{k,n},
\]
where \( \Delta = d/l, \lambda = c/l \) is the wavelength.

Therefore, the covariance matrix can be written as
\[
\mathbf{R}_k = S \mathbf{R} + \sigma^2 I_M,
\]
where
\[
\mathbf{R} = a(\theta) a^H(\theta) \otimes B(\theta, \sigma_\theta),
\]
\( \otimes \) is the Shur-Hadamard product and
\[
B(k,n) = e^{-2[\pi \Delta(k-n) \sigma_\theta \cos\theta]^2}
\]

3. The MUSIC algorithm

The MUSIC algorithm is based on the decomposition of the estimated spatial covariance matrix, \( \hat{\mathbf{R}}_k \), into a signal subspace \( \hat{\mathbf{E}}_s \) and a noise subspace \( \hat{\mathbf{E}}_n \) as follows
\[
\hat{\mathbf{R}}_k = \hat{\mathbf{E}}_s \hat{\mathbf{A}}^H \hat{\mathbf{E}}_s^H + \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^H \sigma^2,
\]
where
\[
\hat{\mathbf{R}}_k = \frac{1}{K} \sum_{n=1}^{K} x(t_n) x^H(t_n),
\]
\( K \) is the number of snapshots, \( \hat{\mathbf{A}}_s \) is a diagonal matrix containing the \( N \) largest eigenvalues of \( \hat{\mathbf{R}}_k \) in a decreasing order, and the columns of \( \hat{\mathbf{E}}_s \) are the corresponding eigenvectors. \( \hat{\mathbf{E}}_n \) is formed by the remaining \( M-N \) eigenvectors. The DOA are estimated by searching one by one for values of \( \theta \) that make
\[
a(\theta) \text{ nearly orthogonal to the noise subspace, i.e.,} \quad \hat{\mathbf{E}}_n^H a(\theta) = 0.
\]

In practice, \( \theta \) is estimated by maximizing the following spectrum (R. O. Schmidt, 1981)
\[
P_{\text{MUSIC}}(\theta) = \frac{1}{a^H(\theta) \hat{\mathbf{E}}_s \hat{\mathbf{E}}_n^H a(\theta)}.
\]

4. The COMET algorithms

In this section, the class of covariance matching estimators (B. Ottersten & al., 1998) is described. COMET estimators are known to provide computationally efficient alternatives to the Maximum Likelihood estimator (T. Trump & al., 1996), whenever the covariance matrix \( \mathbf{R}_k \) can be written explicitly in terms of the set of the parameters \( \eta = [S \sigma^2 \sigma_\theta \theta]^T \). In the presence of a single source, COMET consists in minimizing the following criterion (O. Besson & al., 2000)
\[
J(\eta) = \left\| W^{1/2} (\mathbf{R}_k(\eta) - \hat{\mathbf{R}}_k) W^{1/2} \right\|_F^2
\]
\[
= \text{Tr}\{\mathbf{R}_k(\eta) - \hat{\mathbf{R}}_k W(\mathbf{R}_k(\eta) - \hat{\mathbf{R}}_k) W\}.
\]
where $W$ is a positive definite weighting matrix.

In (T. Trump & al., 1996), it is shown that the covariance matrix of the COMET estimation error, $\hat{\eta} = \eta - \eta$, is given

$$C = E\{\hat{\eta}\hat{\eta}^H\} = H^{-1}QH^{-1}. \quad (11)$$

where

$$H(k, n) = 2Tr \left[ \frac{\partial R_s}{\partial \eta_k} W \frac{\partial R_s}{\partial \eta_n} W \right]$$

and

$$Q(k, n) = \frac{4}{K} Tr \left[ R_s W \frac{\partial R_s}{\partial \eta_k} W W \frac{\partial R_s}{\partial \eta_n} W \right]$$

4.1 The Weighting Least Square estimator

In (M. Viberg, & al. 1991), it is shown that $W = \hat{R}_K^{-1}$ yield consistent estimates. By substituting $W$ in the criterion given by (10), we obtain

$$J(\eta) = \left\| \hat{R}_K^{-1/2} (R_s(\eta) - \hat{R}_K)^{1/2} \right\|_F^2$$

$$= Tr \left\{ (R_s(\eta) - \hat{R}_K)^{1/2} I^2 \right\}$$

The obtained estimator is called the Weighting Least Square (WLS). Since the WLS criterion is quadratic in $S$ and $\sigma^2$ and by differentiating with respect to $S$ and $\sigma^2$, we obtain

$$\hat{S} = \frac{Tr \left[ \hat{R}_K^{-1} \hat{R}_K^{-1} - 2 \hat{R}_K^{-1} \hat{R}_K^{-1} \right]}{Tr \left[ \hat{R}_K^{-1} \hat{R}_K^{-1} - \hat{R}_K^{-1} \hat{R}_K^{-1} \right]}$$

and

$$\hat{\sigma}^2 = \frac{Tr \left[ \hat{R}_K^{-1} \hat{R}_K^{-1} - \hat{R}_K^{-1} \hat{R}_K^{-1} \right]}{Tr \left[ \hat{R}_K^{-1} \hat{R}_K^{-1} - \hat{R}_K^{-1} \hat{R}_K^{-1} \right]}$$

Then the estimates of the nominal DOA and the angular spread can be separated from that of $S$ and $\sigma^2$, as follows

$$[\theta, \sigma^2] = \arg \min \limits_{\theta, \sigma^2} \left\{ (\hat{S} \hat{R}(\theta, \sigma^2) + \hat{\sigma}^2 I) \hat{R}_K^{-1} I \right\}$$

4.2 The Least Square estimator

A less complex form of the criterion is obtained for $W = I$. The corresponding metric is given by

$$J(\eta) = \left\| (R_s(\eta) - \hat{R}_K)^{1/2} \right\|_F^2$$

$$= Tr \left\{ (R_s(\eta) - \hat{R}_K) (R_s(\eta) - \hat{R}_K)^H \right\}$$

The obtained estimator is called the Least Square (LS) estimator. By differentiating with respect to $S$ and $\sigma^2$, we have

$$\hat{\sigma}^2 = \frac{1}{M} Tr \left[ \hat{R}_K \right]^{-1} \hat{\delta}^2$$

and

$$\hat{\delta}^2 = \frac{Tr \left[ \hat{R}_K R \right] - M \hat{\delta}^2}{Tr \left[ \hat{R}_K^2 \right] - M}$$

Substituting equation (15) in (16), we obtain

$$\hat{\delta}^2 = \frac{Tr \left[ \hat{R}_K R \right] - Tr \left[ \hat{R}_K \right]}{Tr \left[ \hat{R}_K^2 \right] - M}$$

The estimates of $\sigma^2$ and $\theta$ can be obtained as follows

$$[\theta, \sigma^2] = \arg \min \limits_{\theta, \sigma^2} \left\{ (\hat{R}_K - \hat{S} \hat{R}(\theta, \sigma^2) - \hat{\sigma}^2 I) \right\}$$

where $\hat{S}$ and $\hat{\sigma}^2$ are respectively given by equations (15) and (17).

5. Cramer-Rao Bound

The Cramer-Rao Bound (CRB) provides a lower bound on the covariance matrix of any unbiased estimator. The CRB on the covariance of the estimated parameter vector $\eta = [S \; \sigma^2 \; \theta]^T$ is given by the Bangs formula (D. Astély & al. 1999)

$$E\{(\eta - \eta)(\eta - \eta)^H\} \geq FIM^{-1},$$

where the $(k, n)$-th element of the Fisher Information Matrix (FIM) is given by

$$FIM(k, n) = K \left\{ \frac{\partial R_s}{\partial \eta_k} W \frac{\partial R_s}{\partial \eta_n} W \right\}$$

6. Simulation results

In this section, we provide simulation results for the COMET estimator as well as a comparison with the MUSIC algorithm and the CRB in terms of RMSE of the DOA and angular spread estimates. Consider a ULA with $M=8$ sensors uniformly separated by a half wavelength and a single source with nominal DOA $\theta=0^\circ$. The distribution of $\theta$ is assumed to be Gaussian and the signal was generated according to (1) with $L=100$ independent scatterers. The signal to-noise ratio (SNR), is defined as $\text{SNR}=S/\sigma^2$. The Root Mean Square error (RMSE) simulation results are based on 500 Monte Carlo simulations. In figures 1 to 6, the MUSIC simulation results appear as asterisks (*). Simulation results of
WLS estimator appear as squares (\(\square\)). Simulation results for the LS estimator appear as circles (\(\circ\)), theoretical LS and the square-root of the CBR are plotted as dotted and solid lines respectively. All values are given in degrees.

The RMSE values of the nominal DOA estimates and the angular spread estimates are respectively plotted in figures 1 and 2 with respect to the number of snapshots for \(\sigma_\theta=5^\circ\) and SNR fixed to 10 dB. We note that simulation results agree with the theoretical ones when the number of snapshots is high. This is due to the fact that the theoretical analysis is valid only for a large number of snapshots. We also notice that MUSIC performance is worse than that of WLS and LS.

In figures 3 and 4, we present the RMSE values of DOA and angular spread estimates when \(\sigma_\theta\) is varied from 1° to 10° for SNR=10 dB and a number a snapshot \(K=100\). For large values of the angular spread, we remark that the WLS and LS simulation results are far from respectively the CRB and LS theoretical analysis (11). This is due to the fact that the theoretical analysis is based on a first order Taylor expansion of the covariance matrix.

In figures 5 and 6, we study the evolution of the RMSE of the DOA and the angular spread estimates with respect to the SNR for \(K=100\) and \(\sigma_\theta=5^\circ\). We notice that COMET offers better performance than MUSIC even for small SNR.
Figure 5. RMSE of $\theta$ with respect to SNR for $M=8$, $K=100$, $\theta = 0^\circ$ and $\sigma_\theta = 5^\circ$.

Figure 6. RMSE of $\sigma_\theta$ with respect to SNR for $M=8$, $K=100$, $\theta = 0^\circ$ and $\sigma_\theta = 5^\circ$.

Conclusion

In this paper, we have investigated the problem of diffuse source localization. The performance of MUSIC and COMET (Covariance Matching Estimator) in terms of the Root Mean Square error of the DOA and angular spread estimates are compared. Simulation results are shown to agree with those predicted by the theoretical analysis. Both WLS and LS are shown to offer better performance than MUSIC.

References


