Performance of Hybrid ARQ Schemes over Multipath Block Fading Channels

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Abstract: The use of Hybrid Automatic Repeat reQuest (HARQ) for error correction has been given wide attention in the High Speed Downlink Packets Access (HSDPA) mode of the UMTS (3GPP TR 25.848 2001). This paper derives a theoretical analysis of the performance of Automatic Repeat reQuest (ARQ) and Type I Hybrid ARQ (HARQ I) over a block multipath Rayleigh fading channel. It is shown that the derived analytical results are in a good agreement with those obtained by computer simulations. Besides, simulation results of type II HARQ (HARQ II) with a code combining strategy is given in the same context.

Keywords: ARQ, Hybrid ARQ, multipath, block fading channels.

1. Introduction

In future packet based wireless communication systems, the downlink will be excessively used for high data rate transmission such in the High Speed Downlink Packets Access (HSDPA) (3GPP TR 25.848 V4.0.0 2001) which is currently being developed as the evolution of the UMTS system to increase the data rate. To achieve a high system reliability it is common to use the Automatic Repeat reQuest (ARQ) protocol in addition to Forward Error Correction (FEC) (El Bahri & al 2004). In ARQ strategies, when an erroneous packet is detected, a retransmission request is sent to the transmitter until an error free reception is observed. The combination of ARQ and FEC reduces the number of retransmissions and yields three types of Hybrid ARQ schemes in which error correction followed by error detection is applied at every received packet. HARQ I scheme discard erroneous packets and send negative acknowledgements (NACK) to the transmitter. The entire packet is then retransmitted. In HARQ II, the erroneous packets are not discarded as in conventional ARQ and HARQ I systems and retransmission contains incremental redundant bits. The receiver combines these redundant bits with bits of the previous transmissions resulting in lower code rates. Finally, in HARQ III schemes, individually transmitted packets are self-decodable and packets are only combined after decoding has been attempted on each of the individual packets. We here emphasize the fact that type III HARQ packets are all coded with a correcting code which is not the case in type II HARQ. The performance of these schemes is well understood for Binary Symmetric Channel (BSC) and AWGN channel (Wang & al 1983) (Kallel 1990 & 1995) (S. Kallel & al 1989). Some studies have been performed for Rayleigh fading channels, modeled as a multistate Markov chain (Zhang & al 1999) (Sato & al 1993) (Liu & al 1997). In (S. Falahati & al. 1998) (J. Hagemauer 1988), HARQ schemes are analyzed assuming a single block fading path. The path is assumed to be constant over each transmitted packet and independent between the different packets. In this paper, we propose to derive a theoretical study of the performance of ARQ and HARQ I over a multipath block fading channel. A perfect Maximum Ratio Combining strategy is assumed. Therefore, path complex gains and delays
are assumed to be known by the Rake receiver which is the conventional matched filter in Direct Sequence Spread Spectrum Systems (DS-SSS). The spreading factor is assumed to be large and the number of users is assumed to be small so that the Inter Symbol Interference (ISI) and the Multi-User Interference (MUI) can be neglected (Boujemâa 2000).

The paper is organized as follows. Section II gives the system model. Sections III and IV derive the performance of ARQ and HARQ I over a multipath block fading channel. Section V recalls the principle of HARQ II with code combining. Section VI compares simulation and theoretical results of ARQ and HARQ I in terms of Block Error Rate (BLER) and Throughput efficiency (Thr). Simulation results of HARQ II with Code Combining are also given. Finally, section VII draws some conclusions.

2. System model

In this section, we describe the channel decoder input for DS-SSS using an ideal Rake receiver. The Rake receiver is assumed to have a perfect knowledge of complex path gains modules \( \{a_l\}_{l=1}^L \), where \( L \) is the number of paths. If path delays are well separated and in the absence of ISI and MUI, the Rake receiver output for symbol \( s_n \) can be written as (Boujemâa 2000)

\[
\hat{s}_n = \sum_{l=1}^{L} a_l^2 \hat{s}_n + w_n, \tag{1}
\]

where \( \hat{s}_n \) is the \( n \)-th transmitted symbols which is dropped from a BPSK constellation and \( w_n \) is a complex Gaussian noise with variance

\[
Var(w_n) = \sum_{l=1}^{L} a_l^2 N_0, \tag{2}
\]

where \( N_0 \) is the two sided Power Spectral density of the complex channel noise.

The wireless link implements the Selective Repeat (SR) protocol for retransmission of erroneous packets with perfect code detection and suitably large buffers at the transmitter and the receiver. Furthermore, we assume an error free feedback channel over which positive (ACK) or negative (NACK) acknowledgements can be sent.

3. ARQ performance analysis

In this scheme, the transmitter sends a packets consisting of \( k \) information bits and \( n_p \) parity bits for error detection. The receiver checks the integrity of the packet by using a code detector. Depending on the outcome of the error detection decoder, an ACK or NACK message is send back to the transmitter. The sender retransmits the packet upon the receipt of a NACK message, and transmits a new packet if an ACK is received. Assuming that complex path gains are constant during a transmitted packet and independent between the different retransmissions, the probability that the number of transmissions is equal to \( i \) conditional on a set, \( \mathbf{B} \), of successive channel draws is

\[
p(tr = i / \mathbf{B}) = [1 - P_{\text{Bloc}}(\mathbf{a}_i)] \prod_{j=1}^{i-1} P_{\text{Bloc}}(\mathbf{a}_j), \tag{3}
\]

where \( \mathbf{B} = (\mathbf{a}_1, ..., \mathbf{a}_i, ...) \), \( \mathbf{a}_j = (\alpha_j, ..., \alpha_L) \), \( \alpha_L \) is the \( l \)-th path gain during the \( j \)-th transmission, \( P_{\text{Bloc}}(\mathbf{a}_j) \) is the conditioned Block Error Probability (BLEP) given by

\[
P_{\text{Bloc}}(\mathbf{a}_j) = 1 - [1 - p(\mathbf{a}_j)]^{k + n_p}, \tag{4}
\]

\( p(\mathbf{a}_j) \) is the conditioned Bit Error Probability (BEP) at the Rake receiver output (Boujemâa 2000) given by

\[
p(\mathbf{a}_j) = \frac{1}{2} \text{erf} \left( \sqrt{\frac{\sum_{l=1}^{L} (\alpha_j^2 E_s)}{N_0}} \right), \tag{5}
\]

where \( E_s \) is the energy of the transmitted symbols.

The average number of transmissions is then deduced by averaging equation (3) over different channel draws as follows

\[
p(tr = i) = \int p(tr = i / \mathbf{B}) f(\mathbf{B}) d\mathbf{B}, \tag{6}
\]

where \( f(\mathbf{B}) \) is the joint Probability Density Function (PDF) of path gains. Assuming that path gains are independent between the different transmission, we have

\[
f(\mathbf{B}) = \prod_{j=1}^{i} f(\mathbf{a}_j). \tag{7}
\]

Substituting (7) in (6), we obtain

\[
p(tr = i) = P_{\text{Bloc}}^{-i} \left[ 1 - P_{\text{Bloc}} \right], \tag{8}
\]

where \( P_{\text{Bloc}} \) is the average BLEP evaluated numerically for a given path gains vector \( \mathbf{a} = (\alpha_1, ..., \alpha_L) \) as follows

\[
P_{\text{Bloc}} = \int \left[ 1 - k - p(\mathbf{a}) \right]^{k + n_p} f(\mathbf{a}) d\mathbf{a}. \tag{9}
\]

We then deduce the average number of transmission (\( Tr \)) as follows

\[
Tr = \sum_{i=1}^{In} i \times p(tr = i) = \frac{1}{1 - P_{\text{Bloc}}}. \tag{10}
\]

Moreover, the throughput efficiency (\( Thr \)), defined as the average number of accepted information bits per transmitted channel symbol, is given by
Th = \frac{1}{Tr} \frac{k}{k + n_p}, \quad (11)

where the factor \( k/(k+n_p) \) is the loss in throughput due to the added parity check bits for error detection.

4. HARQ I performance analysis

In HARQ I schemes, two binary codes \( C_0 \) and \( C_1 \) are respectively used for error detection and error correction. The use of these two codes permits to include parity bits for both error detection and error correction in every transmitted packet. If the number of erroneous bits in a received packet is within the error correction capability of the code, the errors are corrected and the decoded message is declared error free by the receiver. If an uncorrectable error pattern occurs, the packet is detected in error and its retransmission is requested. The transmitter sends the original packet again. In this work, we assume that a convolutional encoder of rate \( R_c = l/v \) and memory \( m \) is used for error correction. At the receiver, a soft input Viterbi decoder is used. The Block error probability \( P_{\text{Bloc}}(\alpha) \) for a given path gains vector \( \alpha \) can be upper bounded as (Kallel & al 1992)

\[ P_{\text{Bloc}}(\alpha) \leq 1 - (1 - P_E(\alpha))^{k+n_p}, \quad (12) \]

where \( P_E(\alpha) \) is the error event probability upper bounded by

\[ P_E(\alpha) = \sum_{d=\text{free}}^{\text{max}} a_d \times P_d(\alpha), \quad (13) \]

\( a_d \) is the number of incorrect paths at distance \( d \) and

\[ P_d(\alpha) = \frac{1}{2} \text{erf}\left( \frac{dE_i}{\sqrt{N_0}} \sum_{i=1}^{L} \alpha_i^2 \right) \quad (14) \]

is the conditioned probability that a wrong path at distance \( d \) is selected (Proakis 1995).

The average BLER is then deduced by averaging (12) numerically as follows

\[ P_{\text{Bloc}} = \int P_{\text{Bloc}}(\alpha) f(\alpha) d\alpha. \quad (15) \]

We then deduce the average number of transmissions and the throughput efficiency as follows (El Bahri & al 2004)

\[ Tr = \frac{1}{1 - P_{\text{Bloc}}}, \quad (16) \]

\[ Thr = \frac{R_c}{Tr} \frac{k}{k + n_p + lm}, \quad (17) \]

where the factor \( k/(k+n_p+lm) \) is the loss in throughput due to the added parity bits for error detection and to the tail of \( lm \) known bits appended to each transmitted sequence to terminate the convolutional code trellis.

5. HARQ II with Code Combining

Type II HARQ scheme with code combining uses a (2,1,m) convolutional code with two generator polynomials \( G_1 \) and \( G_2 \) for error correction (Kallel 1990). When a \( k \) bit message is ready for transmission, it is first encoded into a codeword \( l \) using \( C_0 \). The sequence \( I \) is then encoded with the correction code \( C_1 \) to obtain two sequences \( V_1 \) and \( V_2 \). In code combining strategy, the transmitter alternately sends these two sequences until the most recent received sequence is decoded error-free or if the combination of all received sequences is declared error-free by the channel decoder. The throughput efficiency of HARQ II with code combining is given by

\[ Thr = \frac{1}{Tr} \frac{k}{k + n_p + lm}. \quad (18) \]

In the presence of a multipath fading channel, the average number of transmissions is difficult to derive since we combine packets corresponding to different channel gains. For this reason, only simulation results of HARQ II with code combining are given to evaluate the performance of this scheme over a multipath block fading channel.

6. Numerical and Simulation Results

In this section, we compare simulation and theoretical results of different HARQ schemes in terms of throughput efficiency and bloc error rate. For the sake of illustration, we consider the following values for the parameters. For the ARQ scheme, each packet consists of \( k=940 \) bits and \( n_p=20 \) parity bits for error detection. For HARQ I and II, a (2,1,8) convolutional code with generator polynomials \((561,753)\) is used for error correction and each packet contains \( k=454 \) information bits and \( n_p=18 \) parity bits for error detection.

Figure 1 shows the BLER of an ARQ scheme for a Gaussian and a multipath Rayleigh block fading channel with \( L \) paths having the same average powers. We see that, as much as the diversity order increases, the obtained performance tends to that of the Gaussian channel. The corresponding throughput efficiency is plotted in figure 2. We also see that the theoretical results agree reasonably well with the simulation results for all \( E_b/N_0 \).

Also, we plot the BLER and throughput efficiency of HARQ I in figure 3 and 4 respectively. Once again, the theoretical results are close to the simulation results. We notice that at high \( E_b/N_0 \) the performances improve as the diversity order increases. However, at low \( E_b/N_0 \) the BLER and the throughput efficiency performance degrade as much as the diversity order increases. This is due to the fact that the probability of having a large received energy increases when the diversity order decreases.
Figure 5 gives simulation results for HARQ II with Code Combining in terms of throughput efficiency. We see that the performance of this scheme improves when the number of path increases. Also, by combining the throughput of HARQ II is better than that of ARQ and HARQI for all $E_s/N_0$.

**Conclusion**

In this paper, we have studied the performance of ARQ and HARQ I schemes in terms of BLER and throughput efficiency for a multipath block fading channel. We observed that as much as the diversity order increases, the obtained performance tends to that of the Gaussian channel. The derived analytical results are shown to be in good agreement with those obtained by computer simulations. Finally, Simulation results of HARQ II with code combining have also be given.

**Figure 1.** BLER of ARQ scheme for $k=940$ and $n_p=20$

**Figure 2.** Throughput of ARQ scheme for $k=940$ and $n_p=20$.

**Figure 3.** BLER of HARQ I scheme for $k=454$ and $n_p=18$.

**Figure 4.** Throughput of HARQ I scheme for $k=454$ and $n_p=18$.

**Figure 5.** Throughput efficiency of HARQ II with code combining.
References


