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A Multicriterion Pretopological Approach for Image Segmentation

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Abstract;We present a multicriterion pretopological formalism that allows the mathematical modeling of image segmentation. The choice of pretopology is motivated by the fact that it has less axioms than the topology which facilitates its adaptation to discrete spaces and in particularly image processing. The multicriterion pretopology is based on a multicriterion adherency that allow in a process of agregation to present the segmented image as a partition of all the mulicriterion pretopological closures (or classes of equivalence).

Key Words: Multiciterion Prétopology, segmentation, edge detection

1. Introduction

The mathematical modeling of various problems in image processing (image representation, segmentation,...) has long been an active area of research. Several studies attacked these problems [Ser 82] [Mal 97] [Cha 91] [Dou 01]. Morphological operators are the most common used and several studies are devoted there. In the years 80, several researchers developed some pretopological tools well adapted to discrete spaces and specially to image processing [Aur 85] [Lam 87] [Dur 87]. These research works are motivated by the fact that in problems using a topological approach, some algorithms don't use, in reality, the topological concepts. The results could not be attached to concepts or evaluated in terms of properties that they are susceptible to possess. The pretopology is obtained by the weakening of the topology's axioms. The choice of the pretopology were motivated by the fact that it has less axioms than the topology which facilitates its adaptation to discrete spaces and in particularly image processing [Lam 87]. For example, in a pretopological space, the adherency function is not idempotent, what allows it successive applications on objects and then the mathematical modeling of segmentation by region growing [Mam 01]. In previous work [Mam 01], we used some pretopological operators for the modeling of the image segmentation by region growing and we gave two applications: the extraction of handwriting from check background and edge detection. In this paper, we extend the pretopological formalism to a multicriterion pretopological one and give some nices properties in images segmentation.

In section 2, we expose briefly the notion of pretopology and we introduce the multicriterion pretopological formalism. In section 3, we apply our formalism to image segmentation

2. Multicriterion Pretopology Formalism

In order to weaken the axiomatic of topology, several researchers [Aur 85] [Lam 87] [Dur 87] developed the notion of pretopological structure. G.Duru [Dur 87] spread it in order to study and analyze some structures in

economical and social sciences. The pretopological structures carried their mathematical contributions to the image processing domain for which they represent a tool clearing on computer procedures in total agreement with thetheoretical concepts [Aur 85] [Lam 87] [Mam 01]. Like topology, pretopology can be constructed from the Kuratowski's axioms. Let E be a set (the support of an image for example) and P(E) the set of all subsets of E. An adherency function a(.) is defined on P(E) by: $a : P(E) \to P(E)$ such that:

$$(1) \quad \begin{cases} i) \quad a(f) = f \\ ii) \quad \forall A \subset P(E),\ A \subset a(A) \end{cases}$$

The pretopological space is noted (E,a). The adherency function is not idempotent which allows it for successive applications on images. In previous works [Mam 98] [Mam 01], we show that pretopological closures are best candidates to partition, or segmentation by region growing, images in connected and homogeneous subsets. We introduce for that an adherency function which takes into account the homogeneity between a point and its neighbors. Next, we extend the notion of pretopology to a multicriterion one based on some criterions and we give a formalism of the segmentation by region growing.

Let E (E $\subset Z^2$) be a set (the support of an image for example) and $\Omega = \{C_1, ..., C_p\}$ a set of p criterions defined on E. For x $\in$ E, we define the set

$$(2) \quad B_i(x) = \{y \in V_8(x),\ C_i(x,y) = V\} \quad i=1,....,p$$

where V8(x) designates the set formed by the point x and its eight neighbors and Ci(x,y)=V means that x and y are homogenous related to the criterion Ci. Note that we can use any other structuring elements as V4(x).

Let q an integer such that, $a = p - q + 1 \qquad q \le p$

$$\Gamma_a = \{I \subset \{1,2..., p\} : \|I\| = a\}$$ and

$$B(x) = \left\{ \bigcup_{I \in J} B_j(x),\ J \subset \Gamma_a \right\}$$

can be considered as a prefilter base.

We define the multicriterion adherency a_{pq} by :

$$a_{pq}(A) = \left\{ x \in E, \forall I \subset \Gamma_a,\ \left(\bigcup_{i \in I} B_i(x) \right) \cap A \neq f \right\} \quad (3)$$

The multicriterion pretopological space is noted (E,apq). The adherency function apq is not idempotent which allows it for successive applications on images. This multicriterion pretopology verifies the properties:

$$(4) \quad A \subset B \Rightarrow a_{pq}(A) \subset a_{pq}(B)$$

$$(5) \quad q \le q' \Rightarrow \forall A \subset E,\ a_{pq'}(A) \subset a_{pq}(A)$$

which means that the multicriterion pretopology defined by apq' is more fine that the one defined by apq.

$$(6) \quad x \in a_{pq}(A) \Leftrightarrow B_i(x) \cap A \neq f$$

for at least q criterions from the p one.

Note that we can define others pretopological operators from the multicriterion adherency function:

$$(7) \quad i_{pq}(A) = (a_{pq}(A^c))^c$$ - Interior :

$$(8) \quad fr_{pq}(A) = a_{pq}(A) \cap a_{pq}(A^c)$$ - Frontier:

- Border :

$$(9) \quad b_{pq}(A) = \left\{ x \in A, \forall I \subset \Gamma_a,\ \bigcup_{i \in I} B_i(x) \cap A^c \neq f \right\}$$

- Derivative:

$$(10) \quad d_{pq}(A) = \left\{ x \in A, \forall I \subset \Gamma_a \left(\bigcup_{i \in I} B_i(x) - \{x\} \right) \cap A \neq f \right\}$$

The figure 1 give an idea on the effect of these operators on a handwritten signature S with the adherency a4(A) = { x ∈ E , V4(x) ∩ A ≠∅ } (pretopology defined with no criterion).

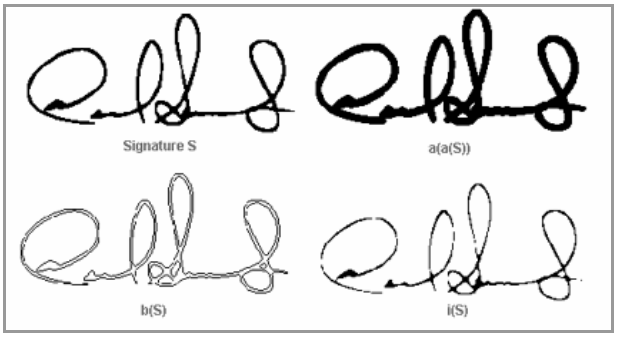


Figure 1. *Effect of pretopological operators on a handwriting signature*

We give now some examples of multi-criterion pretopologies:

Let $\Omega = \{C_1, C_2, C_3\}$ a set of 3 criterions defined on E and

$$B_i(x) = \{y \in V_8(x),\ C_i(x,y) = V\} \quad i=1,2,3.$$

We can define 3 multicriterion pretopologies on the set E as follow:

with 3 criterions from 3: (E, a_{33}) **Pretopology** –

We have in this case: a =3-3+1=1 ;

and the $\Gamma_1 = \{I \subset \{1,2,3\} : \|I\| = 1\} = \{\{1\},\{2\},\{3\}\}$ multicriterion adherency :

$$a_{33}(A) = \{x \in E, \forall i \in \{1,2,3\} : B_i(x) \cap A \neq f\} \quad (11)$$

with 2 criterions from 3: (E, a_{32}) **Pretopology** – a We have in this case : =3-2+1=2 ;

$$\Gamma_2 = \{I \subset \{1,2,3\} : \|I\| = 2\} = \{\{1,2\},\{2,3\},\{1,3\}\}$$

and the multicriterion adherency :

$$a_{32}(A) = \left\{ \begin{array}{l} x \in E : (B_1(x) \cup B_2(x)) \cap A \neq f, \\ (B_1(x) \cup B_3(x)) \cap A \neq f \\ and\ (B_2(x) \cup B_3(x)) \cap A \neq f \end{array} \right\} \quad (12)$$

with 1 criterion from 3: (E, a_{31}) **Pretopology** – a We have in this case : =3-1+1=3 ;

$$\Gamma_3 = \{I \subset \{1,2,3\} : \|I\| = 2\} = \{\{1,2\},\{2,3\},\{1,3\}\}$$

and the multicriterion adherency :

$$a_{31}(A) = \{x \in E : (B_1(x) \cup B_2(x) \cup B_3(x)) \cap A \neq f\} \quad (13)$$

It is clear that for all A $\subset$ E, A $\subset$ a33(A) $\subset$ a32(A) $\subset$ a31(A) and then the multicriterion pretopologies (E,a33), (E,a32) and (E,a31) are in relation of fitness.

Unfortunately, for the pretopology (E, a_{pq}), we have not the interesting property :

$$a_{pq}(A) = \bigcup_{x \in A} a_{pq}(x) \quad (14)$$

which would permit as in the case of a pretopology associated to one criterion [8] to have the pretopological closure of a set A, by the pretopological closures of its elements. This property would permit to engage the process of aggregation by some chosen pixels as germs. To avoid this constraint, we use the inverse adherency a'_{pq} defined by :

$$a'_{pq}(A) = \{x \in E, a_{pq}(x) \cap A \neq f\} \quad (15)$$

that verifies the property:

$$a'_{pq}(A) = \bigcup_{x \in A} a'_{pq}(x) \quad (16)$$

and then permit to engage the process of aggregation by some chosen pixels as germs. The multicriterion pretopology associated is with p-q+1 criterions from p one and permit also the mathematical modelling of the segmentation by region growing. The segmentation is unique and do not depend on the choice of initial germs.

We have also the property :

$$x \in a'_{pq}(x) \Leftrightarrow \exists y \in A : y \in B_i(x) \quad (17)$$

{ for at least q criterions from the p one }.

3. Multicriterion Pretopology and Segmentation by Region Growing

Mathematically, the segmentation of an image is the partition of its support to connected and homogeneous subsets. Multicriterion pretopology allows mathematical modeling of segmentation by region growing. The pretopological closures are good candidates to partition an image because they are connected, homogeneous with regard to the homogeneity criterions and maximal. The purpose of our approach is to lead to the segmented image, by obtaining all the pretopological closures Ei of the image. The pretopological closures of the image support E, will be made by the pretopological closures of its elements. Since the image support is finite, the pretopological closure of the germ is reached after a finite number of iterations. Finally, the pretopological closures are maximal by the fact that for a given pretopological closure Ei, the complementary of Ei does not have any neighbors in Ei or all its neighbors in Ei are not ''good neighbors'' (i.e. don't verify the homogeneity criterions with any element x $\in$ E). An initial pixel or germ, automatically chosen is used to engage the process of aggregation. This iterative aggregation finished by obtaining the pretopological closure of the germ. This later is chosen among the image points not again treaties or aggregates.

The general algorithm is :

1. Chose an initial germ x1 $\in$ E to start the aggregation process.
2. Search the multicriterion pretopological closure F(x1) of x1 ($\exists r : F(x_1) = (a_{pq})^r(x_1)$).
3. Chose another germ x2 in E-F(x1) and search its multicriterion pretopological closure F(x2).
4. If there is a point in $E - \bigcup F(x_i)$ that can be taken as a germ, go to step 1.
5. Finally, present the resulting image as a partition of all the multicriterion pretopological closures (or classes of equivalence).

Next we give an application of our multicriterion pretopology approach, for the task of edge detection, with two criterions (defined in previous works) : a filiformity

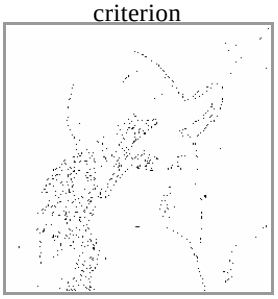
criterion ([Mam 01]) and a criterion of contrast ([Mam 98]).

Original Image

Pretopological closure with the contrast criterion

Pretopological closure with the filiformity criterion

Multicriterion Pretopological closure with the filiformity or the contrast criterion



Multicriterion Pretopological closure with the filiformity and the contrast criterions

Figure 2: Edge detection based on the multicriterion pretopological approach

4. Conclusion

We have presented a multicriterion pretopological approach where the adherency function is defined from some homogeneity criterions. The multicriterion pretopological adherency is non idempotent and permits then the implementation of iterative algorithms. This approach allows the mathematical modeling of the segmentation by region growing where the resulting image is presented as a partition of all the multicriterion pretopological closures. In our future works, we will apply this formalism with criterions based on pretopological measures for the task of off-line handwritten signature verification.

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References

[Aur 85] J.P. Auray, G. Duru , M. Lamure & M. Terrenoire : ‘ Outils Prétopologiques pour Le Traitement Des Images ’ Actes du Colloque " Analyse des Problèmes Décisionnels dans un Environnement Incertain Et Imprécis ", Reims (France) 1985, pp.135-145.

[Cha 91] J.M. Chassery & A. Montavert “ Géométrie Discrète en Analyse d’Images “ Edition Hermes, 1991.

[Dou 01] H.Douzi, D.Mammass & F.Nouboud:" Multiscale Edge Detection and Image Characterization by the Faber-Schauder Multiscale basis" Journal of Mathematical Imaging and Vision, Kluwer Academic Publishers, Vol.14 (2), March 2001, pp.91-101.

[Dur 87] G.Duru " Contribution à l'étude des Structures des Systèmes Complexes dans les Sciences Humaines" Thèse de Doctorat d'Etat, UCB Lyon, France, 1987, 229p.

[Lam 87] M.Lamure & J.J.Milan ” A system of Image Analysis based on a Pretopological Approach “ Intelligent Autonomous System Procceding, 1987, pp.340-345.

[Mal 97] S. Mallat “A Wavelet Tour of Signal Processing “ Associated Press, New York, 1997.

[Mam 01] D. Mammass, S. Djeziri & F. Nouboud ‘ A Pretopological Approach for Image Segmentation and Edge Detection ‘ Journal of Mathematical Imaging and Vision, Kluwer Academic Publishers, Vol. 15, 2001, pp. 169-179.

[Mam 98] D.Mammass, S.Djeziri & F.Nouboud:" Détection de Contours par Approche Prétopologique" Vision Interface’98, 17-20 June 1998, Vancouver, British Columbia, pp.361-363.

[Ser 82] J. Serra “ Image Analysis and Mathematical Morphology “ Academic Press, New York, 1982.