Dispersion Analysis of a Semi-Infinite Periodic Metal Grating on a Grounded Dielectric Slab

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Abstract- The propagation constants of a one dimensional periodic array of identical, equally spaced, thin metallic strips on a grounded dielectric substrate bounded by two magnetic walls are computed using both the averaged boundary conditions and the ray tracing technique. The direction of propagation is arbitrary and is not restricted to directions perpendicular or parallel to the strips of the grating as is usual assumed in the literature. The results have potential applications among others in slotted microstrip antennas and millimeter wave waveguides.

Keywords: Guided waves, averaged boundary conditions, periodic metallic strips, ray tracing

1. Introduction

Guided waves propagating on a periodically corrugated or grated structures are of interest in the study of leaky waves and traveling wave antennas, traveling wave amplifiers, band-pass filters and transmission lines at microwave and millimeter wave frequencies. Structures of this type have, for example, been previously investigated [Bellamine & al.]-[Xiangyin & al.].

In [Bellamine & al.], we studied the properties of guided waves propagating along a metal grating on the surface of a grounded dielectric slab by a new method based on the averaged boundary conditions. The structure consists of a one-dimensional periodic array of identical, equally spaced, thin metallic strips on a grounded dielectric slab. We assumed that the substrate and the ground plane are assumed to be infinite. This article extends to the case when the structure is semi-infinite. So, the objective of our work is to compute propagation constants of a one dimensional periodic array of identical, equally spaced, thin metallic strips on a grounded dielectric substrate bounded by two magnetic walls as shown in Figure 1. The periodicity of the strips is allowed to form any direction with the boundaries. We choose $z$ to be parallel to the infinite length of the strips; we take $x$ upward perpendicular to the substrate surface, and $y$ horizontal and transverse to the strips. The grounded lower surface of the substrate is taken to be the $yz$ plane. The thickness of the dielectric slab is $d$, its permittivity $\varepsilon_r$, and the ground plane is located at $x = 0$. The spacing of magnetic walls is taken to be $w$. We also investigate the effect of the direction of the strips with respect to the magnetic walls (denoted $\theta_0$) as well as the effect of the slot width (denoted $a/p$) on the propagation constants of the structure. We are interested in extracting simple and approximate results. The resonance phenomenon of the structure described above can be examined by considering the reflection and transmission of individual planes inside the structure as they impinge on the edges. Patch fields are obtained using a finite number of rays. Boundary conditions are then applied and from which transverse resonance equations are derived. Then, we proceed to solve these equations for propagation constants as a function of $\theta_0$ defined above for different values of slot widths for different values of existing number of standing waves.
2. Theoretical Development

In MKS units and time dependence \( e^{jwt} \), the generalized averaged boundary conditions at a strip grating lying in the plane interface between two different materials (Bellamine & al.) are that \( E_z \) and \( E_z \) are continuous while:

\[
E_z = \frac{p}{2\pi} \ln(\sec \frac{\pi b}{2p}) \left[ jw\mu_0 (H_y 2 - H_z 1) + \frac{2}{\varepsilon_r + 1} \frac{\partial}{\partial z} (E_x 2 - \varepsilon_r E_x 1) \right]
\]

(1)

where \( a \) is the slot width, \( b \) is the strip width, \( p \) is the period of the structure \( (p = a + b) \). The fields with the subscript 1 are within the dielectric region, while those with subscript 2 are above the strips.

Illustrated in Figure 1 is the \( z \)-component of the plane wave 1 which can be described as:

\[
H_z^{inc} = \alpha_z H_a e^{-j\mu_1(y \sin \phi_1 + z \cos \phi_1)}
\]

(2)

This is the projected incident plane wave in the transverse \( yz \)-plane of the structure. When we speak of rays in the remainder of the article, then we actually mean projections of rays in the \( yz \)-plane. \( H_a \) is a constant field amplitude, \( \phi_1 \) is the angle of the projection of the ray in the transverse plane, and \( k_\mu \) is the wave-number of the incident plane wave in the substrate.

The incident plane wave 1 is reflected at a propagating angle of \( \phi_r \) that can be different from \( \phi_1 \). The wavenumber of the reflected plane wave denoted \( k_\nu \) can also be different from \( k_\mu \). These differences in propagation constants and angles can be attributed to the anisotropy of their corresponding surface waves. The expression of the \( z \)-component of the reflected plane wave 2 is written as:

\[
H_z^{ref} = -\alpha_z H_b e^{-j\mu_1(y \sin \phi_r + z \cos \phi_r)}
\]

(3)

Here \( H_b \) is a constant magnitude field amplitude. The general solution for the \( z \)-component of the magnetic field is:

\[
\vec{H}_z = -\alpha_z (-j\mu_1(y \sin \phi_r + z \cos \phi_r) + H_a e^{-j\mu_1(y \sin \phi_1 + z \cos \phi_1)}
\]

(4)

Now, we enforce the boundary conditions. In our case, we have a \( z \)-directed magnetic wall boundary at \( y = 0 \) and \( y = w \) so that \( H_z(y = 0) = 0 \) and \( H_z(y = w) = 0 \). So that from [3] and the left boundary condition, one has

\[
k_\mu \cos \phi_1 = k_\nu \cos \phi_r
\]

(5)

and

\[
H_a = H_b
\]

(6)

From [3] and the right boundary condition, one gets

\[
(k_\mu \sin \phi_1 + k_\nu \sin \phi_r)w = 2m\pi (m \in Z)
\]

(7)

Where \( m \) represents the number of standing waves. The evaluation of the propagation constant of the structure can be carried out, then, by solving the following set of equations and after some algebraic manipulation

\[
k_\mu \cos \phi_1 = k_\nu \cos \phi_r
\]

(8)

\[
k_\nu^2 = k_\mu^2 - 2k_\mu(\frac{m\pi}{w}) \sin \phi_1 + (\frac{2m\pi}{w})^2
\]

(9)

\[
\sin^2(\theta_0 + \phi_1) = \frac{TE(x_i)}{2k_\nu l_1(1+x_i^2)} + \frac{E_s + \frac{1}{2}k_\nu l_2 TE(x_i)}{E_s + k_\nu l_2(0.5 + \frac{x_i^2}{\varepsilon_r}) TM(x_i)}
\]

(10)

\[
\cos^2(\theta_0 + \phi_1) = \frac{TM(x_i)}{2k_\nu l_1(1+x_i^2)} + \frac{E_s + \frac{1}{2}k_\nu l_2 TE(x_i)}{E_s + k_\nu l_2(0.5 + \frac{x_i^2}{\varepsilon_r}) TM(x_i)}
\]

(11)

\[
= 0
\]

\[
\sin^2(\theta_0 + \phi_1) = \frac{TE(x_r)}{2k_\nu l_1(1+x_r^2)} + \frac{E_s + \frac{1}{2}k_\nu l_2 TE(x_r)}{E_s + k_\nu l_2(0.5 + \frac{x_r^2}{\varepsilon_r}) TM(x_r)}
\]

(12)

\[
\cos^2(\theta_0 + \phi_1) = \frac{TM(x_r)}{2k_\nu l_1(1+x_r^2)} + \frac{E_s + \frac{1}{2}k_\nu l_2 TE(x_r)}{E_s + k_\nu l_2(0.5 + \frac{x_r^2}{\varepsilon_r}) TM(x_r)}
\]

(13)

\[
= 0
\]

Where \( x_1 \), \( x_r \), \( l_1 \), \( l_2 \), \( E_s \), \( E_t \), \( TE \), \( TM \) are respectively defined as follows:

\[
x_1 = \frac{k_\mu}{k_0}
\]

(12)

\[
x_r = \frac{k_\nu}{k_0}
\]

(13)

\[
l_1 = \frac{p}{\pi} \ln(\sec \frac{\pi b}{2p})
\]

(14)
\[ I_2 = \frac{p}{\pi} \ln(\csc \frac{\pi b}{2p}) \]  

(15)

\[ E_s(x) = \sqrt{\varepsilon_r - x^2} \]  

(16)

\[ E_t(x) = \sqrt{x^2 - 1} \]  

(17)

\[ TE(x) = E_t(x) + E_s(x) \cot(E_s k_0 d) \]  

(18)

\[ TM(x) = \varepsilon_r \cot(E_s k_0 d) - \frac{E_s}{E_t} \]  

(19)

The unknowns are \( k_{li}, k_{tr}, \phi_i, \phi_r \). The phase constants are, then, computed by the following relations:

\[ \beta = k_{li} \cos \phi_i = k_{tr} \cos \phi_r \]  

(20)

A simple algorithm can be used to solve this set of nonlinear equations [7], [9]-[11]. It is as follows:

1. Find \( k_{li} \) for each value of \( \phi_i \) from \(-\pi/2\) to \(\pi/2\) incremented by 0.01\(\pi\) using [10]
2. Compute \( k_{tr} \) from [9]
3. Compute \( \phi_r \) from [11]
4. Substitute the computed values of \( k_{tr} \) and \( \phi_r \) respectively from steps 2 and 3 into [7]. If the latter equation is satisfied, then a root is detected. If not, then check for a sign change for consecutive values of \( \phi_i \). In this case, many standard methods (such as Newton’s method) can be used to evaluate the existing root. This algorithm is very simple to implement. It can also solve for all the existing real roots.

3. Numerical results

Figure 2 shows the normalized propagation constant (with respect to the free space propagation \( k_0 \)) on the propagation angle (which is the direction of the fundamental slow surface waves supported by the structure in the grid plane). The following parameters are used: \( \varepsilon_r = 2.2 \), a small electrically think substrate \( (k_0 d = 0.1) \), and a comparable slot width with respect to the substrate height \( (\frac{a}{d} = 1) \). We see that the surface wave is weakly bound to the substrate as the propagation constant is quite close to that of the free space. At a direction parallel to the metallic strips, the normalized propagation constant is near \( \sqrt{\varepsilon_r} \) and for directions perpendicular to the strips, the normalized propagation constant increases without bound. The fields of the grating waves are highly localized to the neighborhood of the grating when the normalized propagation constant exceeds \( \sqrt{\varepsilon_r} \). The surface waves have the normalized propagation constant between 1 and \( \sqrt{\varepsilon_r} \). Figure 3 shows the dependence of the normalized propagation constant on the propagation angle, but now for different values of strips direction \( \theta_0 \). We observed as expected a shift of the results with respect to the case when the strips are not oblique (i.e. when \( \theta_0 = \frac{\pi}{2} \)). Figure 4 shows the dependence of the normalized phase constant \( \beta \) on the direction of the strips \( \theta_0 \) for different values of slot width \( m = 0 \). We observe that \( \beta \) is an increasing function of \( \theta_0 \). We notice that for straight strips - that is when the strips are perpendicular to the magnetic walls - the normalized phase constant is equal to the normalized propagation constant corresponding to a propagating angle parallel to the strips. It can also be seen from Figure 4 the linear variation of \( \beta \) with respect to the slot width for a fixed angle \( \theta_0 \). As an example, the normalized phase constant when the strips are parallel to the magnetic walls goes from 0.75 to 1.61 (a range of 0.86). This range of values is almost the same as that when the strips are perpendicular to the magnetic walls.

We have also carried computations for the case when \( m = 1 \). We observed that a sort of cutoff phenomena since only for a range of \( \theta_0 \) angles that a normalized propagation constant exists. The cutoff phenomena accentuates as the slot width decreases. For example, when \( a/p = 0.5 \), surface waves start to propagate at about 59°. We also noticed that as the slot width increases, the nonlinear dependence, for the same direction of the slots, is more pronounced.

4. Conclusion

The properties of the guided waves propagating along a semi-infinite periodic metal grating on the surface of the grounded dielectric slab are studied. Unlike previous theories, the direction is taken to be arbitrary. We treated the cases for both straight and oblique strips.

In the future, we plan to study the structure when it is finite on all four sides. We will then use the structure as a potential broadband antenna with more bandwidth than that of a regular microstrip antenna. Grating the patch antenna into diagonal slots will enforce the current to flow in the direction of slots. Each slot will resonate at a slightly different frequency. The staggering of resonances will yield wider bandwidth.
References


Figure 1: A semi-infinite bounded diagonally slotted structure

Figure 2: Normalized Dispersion graphs for different slot widths
Figure 3: Normalized Dispersion graphs for different angles

Figure 4: Normalized Dispersion graph