SETIT 2005
3rd International Conference: Sciences of Electronic,
Technologies of Information and Telecommunications
March 27-31, 2005 – TUNISIA

An Efficient Calculation of Surface Waves in Anisotropic Printed Antenna Structures

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Abstract: In this work, the TM and TE modal equations of the guided surface waves supported by a multilayered structure of uniaxially anisotropic dielectrics are efficiently formulated and solved. The proposed method for determining the TM and TE characteristic equations leads to a concise form of these, expressed in terms of a 4x4 matrix multiplication, which is easily implemented. Example of a double-layered structure is used to validate the general expression. Numerical results for the cutoff frequencies of the surface wave modes of a five-layer isotropic structure are presented. Finally, a simple approximate formula for the location of the TM_0 mode is also given.
Key words: Printed antennas, multilayered media, surface waves, TM and TE modal equations.

1 Introduction

The surface wave modes are waves, which result from the existence of an interface between air and the dielectric substrate of a grounded microstrip structure (Soares & al., 1989). The study of surface wave excitation on microstrip antennas is justified for the development of better designs of these antennas. Most related studies formulate the TM and TE modal equations as two transcendental equations whose expressions arise in the denominators of the dyadic Green's function (Pozar, 1987, Fan & al., 1992) Peixeiro and Barbosa (Peixeiro & al., 1992) derived the characteristic mode equations for single and double-layered grounded anisotropic dielectric structures. The procedure followed avoids the calculation of the dyadic Green's function and leads to two homogeneous linear systems, the determinant of which give the TM and TE modal equations. When the number of layers is more than 2, the size of each system becomes large involving an important effort of algebraic operations.

In this paper, a new approach to TM and TE modal equations of stratified planar anisotropic structure is presented. The proposed method provides a number of features both in analytical and numerical phases and leads to a simple 4x4 matrix multiplication, which is easily implemented.

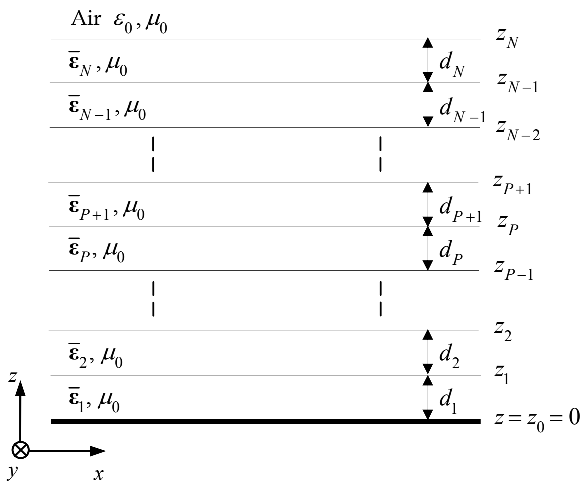

Figure 1. *Cross section of open stratified planar structures*

2 Analysis method

In this section, a new approach to TM and TE modal equations is proposed. The geometry under consideration is depicted in figure 1. It consists of an infinitely large and perfectly conducting plane on which N uniaxial anisotropic dielectrics are superposed in a stratified configuration. Each layer of thickness $d_j = z_j - z_{j-1}$ (j=1,2,...,N) is characterized by the free-space permeability μ_0 and a permittivity tensor of the form

$$\bar{\boldsymbol{\varepsilon}}_j = \varepsilon_0 \,\mathrm{diag}\left[\varepsilon_{xj},\, \varepsilon_{xj},\, \varepsilon_{zj}\right] \qquad (1)$$

ε_0 is the free-space permittivity and diag stands for the diagonal matrix with the diagonal elements appearing between the brackets. Equation [1] can be specialized to the isotropic substrate by allowing $\varepsilon_{xj}=\varepsilon_{zj}=\varepsilon_{rj}$. The ambient medium is air with constitutive parameters μ_0 and ε_0. Assuming an $e^{i\omega t}$ time variations and starting from Maxwell's equations in the Fourier transform domain, we can show that the transverse fields inside the j^{th} layer $(z_{j-1} \langle z \langle z_j)$ can be written in terms of the longitudinal components $\tilde{E}_z$ and $\tilde{H}_z$ as (Pozar, 1987):

$$\tilde{\mathbf{E}}(\mathbf{k}_s, z) = \begin{bmatrix} \tilde{E}_x(\mathbf{k}_s, z) \\ \tilde{E}_y(\mathbf{k}_s, z) \end{bmatrix} = \bar{\mathbf{F}}(\mathbf{k}_s) \cdot \begin{bmatrix} \frac{i}{k_s} \frac{\varepsilon_{zj}}{\varepsilon_{xj}} \frac{\partial \tilde{E}_z(\mathbf{k}_s, z)}{\partial z} \\ \frac{\omega \mu_0}{k_s} \tilde{H}_z(\mathbf{k}_s, z) \end{bmatrix} = \bar{\mathbf{F}}(\mathbf{k}_s) \cdot \mathbf{e}(\mathbf{k}_s, z) \qquad (2)$$

$$\tilde{\mathbf{H}}(\mathbf{k}_s, z) = \begin{bmatrix} \tilde{H}_y(\mathbf{k}_s, z) \\ -\tilde{H}_x(\mathbf{k}_s, z) \end{bmatrix} = \bar{\mathbf{F}}(\mathbf{k}_s) \cdot \begin{bmatrix} \frac{\omega \varepsilon_0 \varepsilon_{zj}}{k_s} \tilde{E}_z(\mathbf{k}_s, z) \\ \frac{i}{k_s} \frac{\partial \tilde{H}_z(\mathbf{k}_s, z)}{\partial z} \end{bmatrix} = \bar{\mathbf{F}}(\mathbf{k}_s) \cdot \mathbf{h}(\mathbf{k}_s, z) \qquad (3)$$

where $\mathbf{e}$ and $\mathbf{h}$ are, respectively, the transverse electric and magnetic fields in the (TM,TE) representation, and

$$\bar{\mathbf{F}}(\mathbf{k}_s) = \frac{1}{k_s} \begin{bmatrix} k_x & k_y \\ k_y & -k_x \end{bmatrix} = \bar{\mathbf{F}}^{-1}(\mathbf{k}_s) \qquad (4)$$

where $\mathbf{k}_s = \hat{\mathbf{x}}\, k_x + \hat{\mathbf{y}}\, k_y$ and $k_s = |\mathbf{k}_s|$. By substituting the expressions of the components $\tilde{E}_z$ and $\tilde{H}_z$ (Pozar, 1987) into [2] and [3], we obtain

$$\mathbf{e}(\mathbf{k}_s, z) = e^{-i\bar{\mathbf{k}}_{zj} z} \cdot \mathbf{A}_j(\mathbf{k}_s) + e^{i\bar{\mathbf{k}}_{zj} z} \cdot \mathbf{B}_j(\mathbf{k}_s) \qquad (5)$$

$$\mathbf{h}(\mathbf{k}_s, z) = \bar{\mathbf{g}}_j(\mathbf{k}_s) \cdot \left[e^{-i\bar{\mathbf{k}}_{zj} z} \cdot \mathbf{A}_j(\mathbf{k}_s) - e^{i\bar{\mathbf{k}}_{zj} z} \cdot \mathbf{B}_j(\mathbf{k}_s) \right] \qquad (6)$$

In equations [5] and [6], $\mathbf{A}_j$ and $\mathbf{B}_j$ are two-component unknown vectors and

$$\bar{\mathbf{g}}_j(\mathbf{k}_s) = \mathrm{diag}\left[\frac{\omega \varepsilon_0 \varepsilon_{xj}}{k^e_{zj}},\, \frac{k^h_{zj}}{\omega \mu_0} \right] \qquad (7a)$$

$$\bar{\mathbf{k}}_{zj} = \mathrm{diag}\left[k^e_{zj},\, k^h_{zj} \right] \qquad (7b)$$

k^e_{zj} and k^h_{zj} are, respectively, propagation constants for TM and TE waves in the j^{th} layer (Pozar, 1987). By writing [5] and [6] in the planes $z = z_{j-1}$ and $z = z_j$, and by eliminating the unknowns $\mathbf{A}_j$ and $\mathbf{B}_j$, we obtain the matrix form

$$\begin{bmatrix} \mathbf{e}(\mathbf{k}_s, z_j^-) \\ \mathbf{h}(\mathbf{k}_s, z_j^-) \end{bmatrix} = \bar{\mathbf{T}}_j \cdot \begin{bmatrix} \mathbf{e}(\mathbf{k}_s, z_{j-1}^+) \\ \mathbf{h}(\mathbf{k}_s, z_{j-1}^+) \end{bmatrix} \qquad (8)$$

with

$$\bar{\mathbf{T}}_j = \begin{bmatrix} \bar{\mathbf{T}}_j^{11} & \bar{\mathbf{T}}_j^{12} \\ \bar{\mathbf{T}}_j^{21} & \bar{\mathbf{T}}_j^{22} \end{bmatrix} = \begin{bmatrix} \cos \bar{\boldsymbol{\theta}}_j & -i\, \bar{\mathbf{g}}_j^{-1} \cdot \sin \bar{\boldsymbol{\theta}}_j \\ -i\, \bar{\mathbf{g}}_j \cdot \sin \bar{\boldsymbol{\theta}}_j & \cos \bar{\boldsymbol{\theta}}_j \end{bmatrix}, \bar{\boldsymbol{\theta}}_j = \bar{\mathbf{k}}_{zj}\, d_j \qquad (9)$$

which combines $\mathbf{e}$ and $\mathbf{h}$ on both sides of the j^{th} layer as input and output quantities. The matrix $\bar{\mathbf{T}}_j$ is the matrix representation of the j^{th} layer in the (TM,TE) representation. The continuity equations for the tangential field components are

$$\mathbf{e}(\mathbf{k}_s, z_j^-) = \mathbf{e}(\mathbf{k}_s, z_j^+),\ j = 1, 2, ..., N \qquad (10)$$

$$\mathbf{h}(\mathbf{k}_s, z_j^-) = \mathbf{h}(\mathbf{k}_s, z_j^+),\ j = 1, 2, ..., N \qquad (11)$$

Using equations [8], [10] and [11], cascading the matrices by simple multiplication yields

$$\begin{bmatrix} \mathbf{e}(\mathbf{k}_s, z_N^+) \\ \mathbf{h}(\mathbf{k}_s, z_N^+) \end{bmatrix} = \bar{\boldsymbol{\Gamma}} \cdot \begin{bmatrix} \mathbf{e}(\mathbf{k}_s, 0^+) \\ \mathbf{h}(\mathbf{k}_s, 0^+) \end{bmatrix} \qquad (12)$$

where

$$\bar{\boldsymbol{\Gamma}} = \prod_{j=N}^{1} \bar{\mathbf{T}}_j \qquad (13)$$

The transverse electric field must necessarily be zero on a perfect conductor, so that for the perfectly conducting ground plane we have

$$\mathbf{e}(\mathbf{k}_s, 0^-) = \mathbf{e}(\mathbf{k}_s, 0^+) = \mathbf{e}(\mathbf{k}_s, 0) = \mathbf{0} \qquad (14)$$

In the unbounded air region ($z_N \langle z \langle \infty$ and $\varepsilon_x = \varepsilon_z = \varepsilon_r = 1$), the electromagnetic field given by equations [5] and [6] should vanish at $z \to +\infty$, accordind to Sommerfeld's condition of radiation, which yields

$$\mathbf{h}(\mathbf{k}_s, z_N^+) = \bar{\mathbf{g}}_0(\mathbf{k}_s) \cdot \mathbf{e}(\mathbf{k}_s, z_N^+) \qquad (15)$$

where $\overline{\mathbf{g}}_0(\mathbf{k}_s)$ can be easily obtained from the expression of $\overline{\mathbf{g}}_j(\mathbf{k}_s)$ given in equation [7a] by allowing $\varepsilon_{xj} = \varepsilon_{zj} = 1$. Combining equations [12], [14] and [15], we obtain the characteristic equations for the TM and TE surface wave modes as :

$$\overline{\mathbf{g}}_0 \,.\, \overline{\mathbf{\Gamma}}^{12} - \overline{\mathbf{\Gamma}}^{22} = \mathbf{0} \qquad (16)$$

Due to the simple formulation, there are not restrictions on the number of layers ; the presence of an arbitrary number of layers is easily included in the matrix product $\overline{\mathbf{\Gamma}}$. For structures with several dielectric layers (more than 2 layers), equation [16] is best to be evaluated numerically.

3 Results and discussion

To validate the technique proposed in section 2, Equation [16] is used to derive the TM and TE modal equations of a substrate-superstrate configuration (Bouttout & al., 2000). The substrate is considered anisotropic while the superstrate is isotropic. With the present formalism, the TM and TE modal equations can be obtained analytically in an easy way. These equations can be expressed as

$$\cos(k_{z2} d_2)\left[\varepsilon_{x1} k_{z0} \cos(k^e_{z1} d_1) + \mathrm{i}\, k^e_{z1} \sin(k^e_{z1} d_1)\right] +$$
$$\mathrm{i} \sin(k_{z2} d_2)\left[\frac{\varepsilon_{x1}}{\varepsilon_{r2}} k_{z2} \cos(k^e_{z1} d_1) + \mathrm{i}\, \varepsilon_{r2} \frac{k_{z0} k^e_{z1}}{k_{z2}} \sin(k^e_{z1} d_1)\right] = 0 \qquad (17a)$$

$$\cos(k_{z2} d_2)\left[k^h_{z1} \cos(k^h_{z1} d_1) + \mathrm{i}\, k_{z0} \sin(k^h_{z1} d_1)\right] +$$
$$\mathrm{i} \sin(k_{z2} d_2)\left[\frac{k_{z0} k^h_{z1}}{k_{z2}} \cos(k^h_{z1} d_1) + \mathrm{i}\, k_{z2} \sin(k^h_{z1} d_1)\right] = 0 \qquad (17b)$$

where k_{z0} is the propagation constant in the air region (Pozar, 1987). Equations [17a] and [17b] are, respectively, characteristic equations for TM and TE waves. It is easy to check that the TM and TE modal equations given in equations [17a] and [17b] are the same to those that can be deduced from the expression of the dyadic Green's function shown in (Bouttout & al., 2000, Eq. [14]). This validates our theory presented in section 2.

The knowledge of the cutoff frequencies of the surface wave modes allows to predict the propagating modes supported by the structure at a given operating frequency. The cutoff frequencies of the TE_1 and TM_1 surface wave modes for the five-layer isotropic structure studied in (Revankar & al., 1991) are depicted in table 1. The operating frequency of the considered structure ranges from 2.5 to 4 GHz, consequently, only the TM_0 mode with zero cutoff frequency is excited. Since the TM_0 mode is always excited, it is a matter of interest to derive simple formula to predict the location of the corresponding wavenumber. The smoothness of the proposed technique allows to find closed form approximate formula for this mode in the limiting case of electrically small dielectrics. Its location is estimated by the following formula:

$$k_{sp} \approx \left[1 + \frac{1}{2}\left[k_0 \sum_{j=1}^{N} d_j \left(1 - \frac{1}{\varepsilon_{zj}}\right)\right]^2\right] k_0 \,,\, k_0^2 = \omega^2 \varepsilon_0 \mu_0 \qquad (18)$$

Note that in the case of a single layer isotropic dielectric slab ($N=1$, $\varepsilon_{xj} = \varepsilon_{zj} = \varepsilon_r$), equation [18] is reduced to the simple expression given in (Chew & al., 1980). With the aim of cheking the accuracy of formula [18] for thin dielectrics, we compare in table 2 the results computed from equation [18] with those obtained from the numerical solution of equation [16] for the five-layer isotropic structure investigated in (Revankar & al., 1991). The reported data are normalized with respect to k_0. The results of table 2 clearly indicate that formula [18] offers a good accuracy.

Table 1. *Cutoff frequencies of the two first wave modes of an isotropic five-layer structure;* $\varepsilon_{r1} = 2.33$, $\varepsilon_{r2} = 1$, $\varepsilon_{r3} = 2.45$, $\varepsilon_{r4} = 1$, $\varepsilon_{r5} = 2.2$, $d_1 = 0.158$ cm, $d_3 = 0.0762$ cm, $d_5 = 0.0508$ cm.

Surface wave modes	Cutoff frequencies (GHz)		
	$d_2 = 0.05$ cm $d_4 = 0.05$ cm	$d_2 = 0.3$ cm $d_4 = 0.6$ cm	$d_2 = 0.5$ cm $d_4 = 1.16$ cm
TE_1	20.9890	14.6349	11.9893
TM_1	41.6298	28.1435	22.7030

Table 2. *Comparison of exact wavenumbers and approximate formula results of an isotropic five-layer structure;* $\varepsilon_{r1} = 2.33$, $\varepsilon_{r2} = 1$, $\varepsilon_{r3} = 2.45$, $\varepsilon_{r4} = 1$, $\varepsilon_{r5} = 2.2$, $d_1 = 0.158$ cm, $d_3 = 0.05$ cm, $d_3 = 0.0762$ cm, $d_4 = 0.05$ cm, $d_5 = 0.0508$ cm.

Operating frequency (GHz)	Formula [18]	Numerical solution of [16]
2.5	1.0036418	1.0036508
3.25	1.0061547	1.0061791
4	1.0093231	1.0093756

Conclusion

Based on a matrix representation of each dielectric layer, a new approach to TM and TE modal equations of the guided waves supported by a stratified planar structures has been presented. It leads to a clear and easy-to-apply formalism with block matrices which reduces the analytical or numerical procedure to simple matrix multiplications. For the substrate-superstrate configuration, the simplicity to obtain the analytical solutions with this new formalism has been shown. Numerical results for the cutoff frequencies of the surface wave modes of a five-layer isotropic structure have been presented. A simple approximate formula for the location of the TM_0 mode is given. The proposed technique can be used in CAD to predict the propagating surface waves in stratified planar structures, it can be also used for seeking the singularities of the integrands arising in the moment method solution of printed antennas.

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