Multi-frequency Electrical Impedance Tomography: Image Reconstruction Using Complex Sensitivity Matrix with Finite Element Methods

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Abstract: Electrical Impedance Tomography (EIT) is a non invasive imaging technique with widespread applications in medicine and industry. It aims to image the conductivity distribution within an object by making electrical measurements on the surface of the volume.

Single frequency electrical impedance tomography offers many positive attributes which are now being exploited clinically, but there are also problems which have been identified. Multi-frequency electrical impedance tomography has been proposed to solve this problem by making impedance measurements over a range of frequencies. The reason for this is that the impedance of biological tissues varies with frequency, and the variations are different from tissue to tissue. It is therefore possible, in principle to make static image (i.e. images of the distribution of frequency dependence of the impedance within the body) by measuring tissue impedance over a range of frequencies.

In this paper new method concerning multi-frequency electrical impedance tomography have been developed.

In the this new developed method; current density and finite element method have been used for the derivation of the complex sensitivity matrix. The new developed method was applied in some models and acceptable results were found. By using these method it possible to produce improvement and fast image reconstructions.

Key words: Multi-frequency EIT, current density, finite element

INTRODUCTION

Electrical Impedance Tomography (EIT) is a non invasive imaging technique with widespread applications in medicine and industry. It aims to image the conductivity distribution within an object by making electrical measurements on the surface of the volume. Some of the potential advantages of this technique are that[BAR88][BAR90a][BAR90b]:

(a) It offers the possibility of an inexpensive non-harmful imaging technique capable of being used in the intensive care environment for a range of physiological measurements.

(b) It will be capable of characterizing tissues in a novel physiological way which should enable diseased tissues or abnormal function to be identified, and

(c) It should be capable of imaging fast dynamic activity within the human body because data can be acquired rapidly.

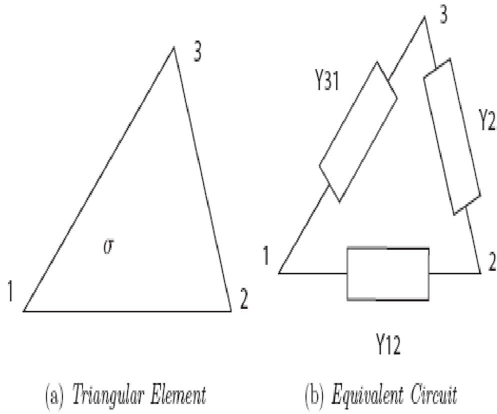
Electrical impedance tomography at single frequency allows images of temporal changes in tissue impedance to be produced. Production of static images using data from human subjects is difficult. The main reason for this is that body shape and electrode position are more important determinants of transfer impedance measurements than is the distribution of tissue impedance.

Multi-frequency EIT offers the potential for making static images by producing images of changes in tissue impedance with frequency. Furthermore, by considering both real and imaginary parts of the conductivity, all information contained in the tissue is obtained.

In this paper new method concerning multi-frequency electrical impedance tomography have been developed. these method it possible to produce improvement and fast image reconstructions.

1. Image Reconstruction

Discretizes the medium under analysis into a finite number of elements collectively called a finite element mesh. Within each element the field variable is approximated by simple functions that are defined only within the individual element. The approximating functions (sometimes called interpolation or shape functions) are defined in terms of the values of the field variables at specified points on the element called nodes. Most EIT work uses linear shape functions in which all nodes lie on the element boundaries where adjacent elements are connected. Higher order shape functions will have interior nodes. In this case the finite element model for EIT is equivalent to a linear electrical network as shown in figure 1[SHA1996].



(a) Triangular Element (b) Equivalent Circuit

Figure 1. *Equivalent circuit of triangular element Triangular element and (b) Equivalent circuit*

From the figure 1 the conductance Y_{ij}, between node i and node *j* is determined by the triangle-to-network conversion as

$$Y_{ij} = \frac{\sigma}{2A}\left(b_i b_j + c_i c_j\right),\ (i \neq j) \tag{1}$$

Where

$b_1 = y_2 - y_3$, $b_2 = y_3 - y_1$, $b_3 = y_1 - y_2$,

$c_1 = x_3 - x_2$, $c_2 = x_1 - x_3$, $c_3 = x_2 - x_1$.

Where $(x_i, y_j), (i = 1,2,3)$ denotes a coordinate of each node, A indicates the area of an element and σ is the element conductivity which is assumed to be constant over the element.

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \quad or \quad Y^e U^e = I^e \tag{2}$$

With

$Y_{11} = -Y_{12} - Y_{13}$, $Y_{22} = -Y_{21} - Y_{23}$, $Y_{33} = -Y_{31} - Y_{32}$, $Y_{ij} = Y_{ji}$, for i,j = 1,2,3

Where:

$u_i (i = 1,2,3)$ are the nodal potentials , $i_i (i = 1,2,3)$ is the current which flows in the i^{th} node and superscript e refers to element e.

Successful imaging depends on the amount of information we acquire from the boundary measurements and the method of extracting information, i.e. the reconstruction algorithm.

From Poisson's equation we assume the boundary condition as following:

$$-\sigma \frac{\partial v}{\partial n} = \begin{cases} J & on \quad the \quad electrodes \\ 0 & on \quad the \quad body \quad air \quad interface \end{cases} \tag{3}$$

where J is the current density , $\frac{\partial v}{\partial n}$ is the potential derivative normal to the surface and σ is the conductivity using finite element method.

For triangle element we can express the current density as [SOL2006]:

$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{a} & 0 \\ 0 & 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{4}$$

$$Y_{ij} = \frac{\sigma}{2A}\left(b_i b_j + c_i c_j\right) \tag{5}$$

$$S^* = V_d{}^* . V_r{}^* \tag{6}$$

where *d* and *r* are drive and receive electrodes.

$$S^* = \alpha S + j\beta S \tag{7}$$

where

$$\alpha = \frac{{\sigma_R}^2}{({\sigma_R}^2 + {\sigma_I}^2)} \tag{8}$$

$$\beta = -2\frac{\dfrac{{\sigma_R}^3}{{\sigma_I}^3}}{\left[\dfrac{{\sigma_R}^2}{{\sigma_I}^2} + 1\right]^2} \tag{9}$$

Where σ_R and σ_I are the real and imaginary parts of the complex conductivity. The boundary voltage measurements is given by

$$\sigma^* = (S^*)^{-1} g^* \tag{10}$$

To normalize change in conductivity to normalized change in boundary complex from

$$\sigma_n{}^* = (F^*)^{-1} g_n{}^* \tag{11}$$

Where F^* is the normalized complex sensitivity matrix, and g_n^* is the normalized voltage measurements.

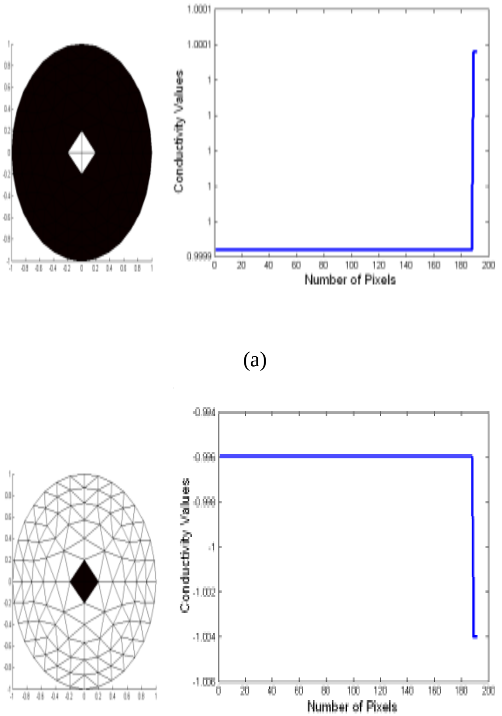
Conductivity image reconstruction can generally be classified into two distinct types: those which seek a full reconstruction of the conductivity distribution, using iterative methods to take account of the non-linearty of the inverse problem, and those which linearize the reconstruction.

2. Results

The results are obtained from two new methods for multi-frequency electrical impedance tomography image reconstruction. It has been used a finite element model consisting of 192 elements. One frequency is used as a reference for the next one so that only the differences in frequency response of the tissue are imaged.

The sensitivity matrix which has been used for the new developed method were carried out using 16 interleave drive and receive electrodes. The dimension of the sensitivity matrix is 64 X 192 it has been chosen the size of the object 0.1.

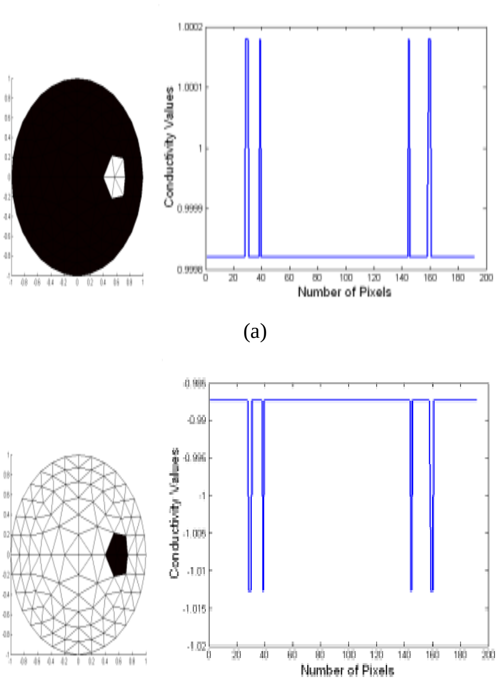
The new developed method shows improvement in quality of image reconstruction than the previous methods. This is because the conductivity values are more affected by the real and imaginary parts of the sensitivity matrix than by the position and size of the object.



(a)

(b)

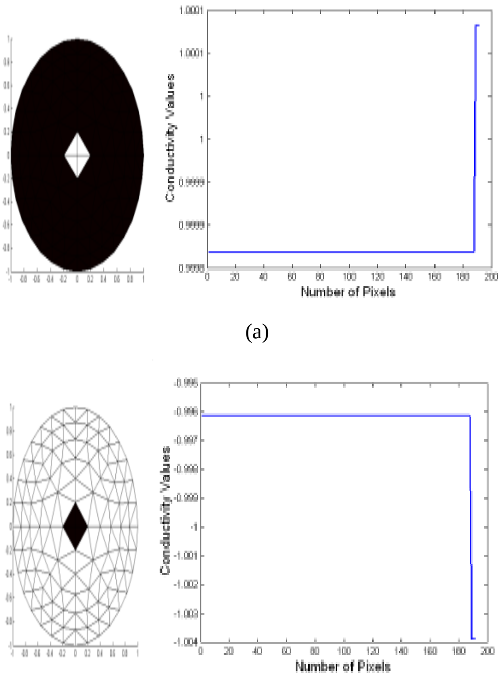
Figure 2. *Image reconstruction of the object at the center using frequency 9.6 KHz: (a) real part of conductivity, (b) imaginary part of the conductivity*



(a)

(b)

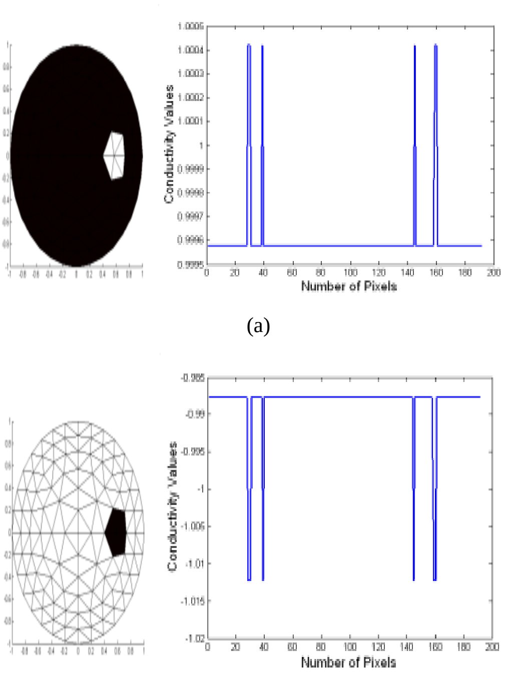
Figure 3. *Image reconstruction of the object at the boundary using frequency 9.6 KHz :(a)real part of conductivity, (b) imaginary part of the conductivity*



(a)

(b)

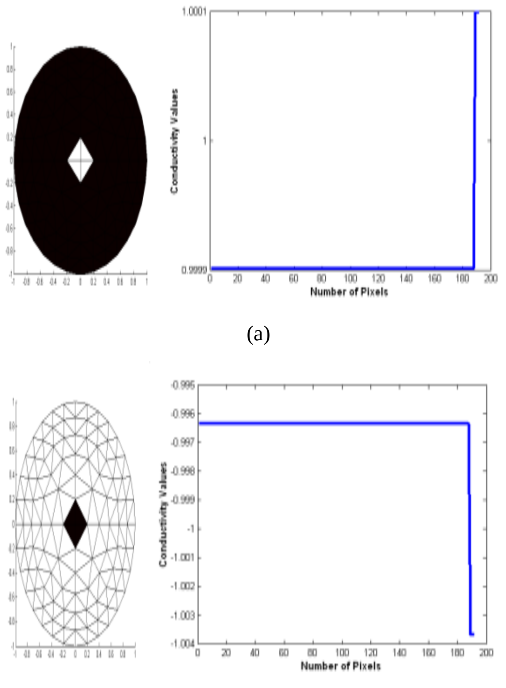
Figure 5. *Image reconstruction of the object at the center using frequency 76.8 KHz: (a) real part of conductivity, (b) imaginary part of the conductivity*



(a)

(b)

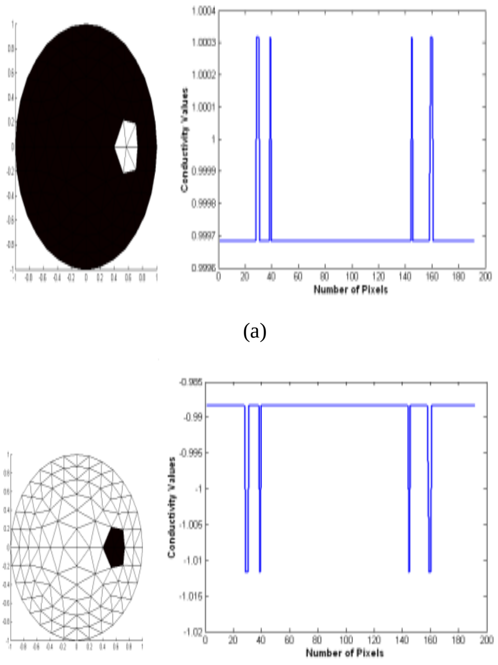
Figure 4. *Image reconstruction of the object at the boundary using frequency 76.8 KHz: (a) real part of conductivity, (b) imaginary part of the conductivity*



(a)

(b)

Figure 6. *Image reconstruction of the object at the center using frequency 614.4 KHz: (a) real part of conductivity, (b) imaginary part of the conductivity*



(a)

(b)

Figure 7. *Image reconstruction of the object at the boundary using frequency 614.4 KHz: (a) real part of conductivity, (b) imaginary part of the conductivity.*

3. Conclusion

Electrical Impedance Tomography (EIT) is a non invasive imaging technique with widespread applications in medicine and industry. It aims to image the conductivity distribution within an object by making electrical measurements on the surface of the volume.

Electrical impedance tomography at single frequency allows images of temporal changes in tissue impedance to be produced. Production of static images using data from human subjects is difficult. The main reason for this is that body shape and electrode position are more important determinants of transfer impedance measurements than is the distribution of tissue impedance.

Multi-frequency EIT offers the potential for making static images by producing images of changes in tissue impedance with frequency. Furthermore, by considering both real and imaginary parts of the conductivity, all information contained in the tissue is obtained.

In this paper new method concerning multi-frequency electrical impedance tomography have been developed.

In the this new developed method; complex sensitivity and finite element method have been used for the image reconstruction.

The new developed method was applied in some models and it has been noted the following:

(i) acceptable results and image improvement were found.

(ii) the image reconstruction which obtained from the new developed method is clear and has high value of complex conductivity in the center and in the boundary for the different frequencies.

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