Nonparametric Bayesian Estimation Structures in the Wavelet Domain of Multiple Noisy Image Copies

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***Abstract*—This paper addresses the recovery of an image from its multiple noisy copies using a nonparametric Bayesian estimator in the wavelet domain. Boubchir et al have proposed a prior statistical model based on the α-stable densities adapted to capture the sparseness of the wavelet detail coefficients. They used the scale mixture of Gaussians theorem as an analytical approximation for α-stable densities, which is not known in general, in order to obtain a closed-form expression of their Bayesian denoiser. Since the proposed estimator has worked well for one copy of corrupted image, we consider its extension to multiple copies in this paper. So, our contribution is to design two fusion structures based on the Bayesian denoiser and the traditional averaging operation, in order to combine all multiple noisy image copies to recover the noise free image. Because of the nonlinearity of the Bayesian denoiser, averaging then Bayesian denoising or Bayesian denoising then averaging will produce different estimators. We will demonstrate the effectiveness of our Bayesian denoiser fusion structures compared to other denoising approaches. Better performance comes at the expense of higher complexity.**

***Index Terms*—Denoising, Bayesian estimation, α-stable distribution, multiple noisy copies, wavelets.**

I. Introduction

Due to the imperfection of image acquisition systems and transmission channels, images are often corrupted by noise. This degradation leads to a significant reduction of image quality and then makes more difficult to perform high-level vision tasks such as recognition, 3-D reconstruction, or scene interpretation. In most cases, this corruption is commonly modeled by a zero-mean additive white Gaussian random noise leading to the following additive degradation model:

$$g = x + \epsilon \tag{1}$$

where g, x and ϵ represent respectively the noisy observed image (of size M pixels), the clean image and the corrupting additive white stationary Gaussian noise (AWGN) with variance σ^2. The problem of recovering x from g is usually known as a denoising problem. This problem is a typical instance of an inverse problem where the solution must consider prior knowledge of the distribution of x. Hence, the prior distribution of natural images or of any other specific class of images plays a key role in any denoising approach. A common approach for modeling the statistical prior of natural images is to estimate their statistical distribution in a transform domain. This is often implemented using some type of wavelet transform. In this context, many researchers have investigated, over the last decade, wavelet-based denoising estimators [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12]. A key underlying characteristic in many of these estimators is the ability of wavelet functions to capture effectively the local features of the processes being modeled.

Hence, in the field of statistics for example, wavelets have been used primarily to deal with problems of a nonparametric character, such as those arising frequently in the context of regression analysis, or when estimating functions such as densities, spectral densities or hazard rates. Most of the wavelet-based denoising estimators have been developed based on Donoho & Johnstone's work [1]. Since then, various Bayesian approaches for nonlinear wavelet shrinkage estimators have been developed recently, and various priors have been proposed to model the statistical behavior of the noiseless wavelet coefficients. These estimators impose a prior distribution on wavelet coefficients designed to capture the sparseness of the wavelet expansions. Then the image is estimated by applying a Bayesian rule to the resulting posterior distribution of the wavelet coefficients. Simoncelli et al. [13] developed non-linear estimators, based on formal Bayesian theory, which outperform classical linear processors and simple thresholding estimators in removing noise from visual images. They used a generalized Laplacian model for the subband statistics of the signal and developed a noise-removal algorithm, which performs a "coring" operation to the data. In recent works [14], [15], it has been shown that α-stable distributions, a family of heavy-tailed densities, are sufficiently flexible and rich to appropriately model wavelet coefficients of images. However, the derived Bayesian estimator proposed in [14], has no closed analytical form in general situation. More recently, in [16], [17], [18], [20], the Bessel K forms (BKF) family has been successfully proposed in wavelet-based Bayesian denoising. In that work, a closed-form expression of the L_2-loss Bayesian shrinkage rule associated with the BKF prior was proposed. In [21], [22], [23], Boubchir et al proposed a prior statistical model based on the α-stable densities adapted to capture the sparseness of the wavelet detail coefficients. Specifically, they used the finite mixture of Gaussians as a fast and numerically stable analytical approximation for α-stable densities in order to obtain closed-form expressions for the Bayesian denoiser. In many applications there are multiple copies of the same or similar images, thus it is interesting to extend the Bayesian denoiser to multiple corrupted copies, in order to obtain the most noise-free copy possible. We address this issue in this paper. So, we propose two approaches based on the Bayesian denoiser and averaging operation, to combine the available noisy copies. Because of the nonlinearity of the Bayesian denoiser, averaging then Bayesian denoising or Bayesian denoising then averaging produce different estimators. One novel aspect of this work is that here the Bayesian denoiser and averaging operation are combined to be used in two parallel fusion structures, to enhance the quality of the recovered image, especially in low SNRs. As will be shown, simulation results demonstrate that the denoising performance can be improved significantly by the developed approaches. The later are better suited for low SNRs and are highly efficient, and inherently robust. The remainder of this paper is organized as follows: The proposed denoising structures are described in Section II. In Section III, we present a set of simulation results and comparisons with existing denoising techniques. Finally, conclusions are drawn in Section IV.

II. Bayesian Denoising Algorithm For Multiple Noisy Copies

A. Wavelet-based Bayesian Denoising

Let $x = \{x_{ij}\}$ denote the matrix of the original image to be recovered. The signal has been transmitted over a Gaussian additive noise channel K times, and at the receiver we have K copies of noisy observations,

$$g(k) = x + \epsilon^{(k)}, k = 1, \dots, K \tag{2}$$

For the k^{th} copy, $\epsilon^{(k)}$ are iid Gaussian $N(0, \sigma^2)$ where σ^2 is the noise variance of the k^{th} copy. The noise samples between different copies are assumed independent. The recovery of the image is done in the orthogonal wavelet transform domain (the readers are referred to standard wavelet literature such as [24], [25] for details of the two-dimensional dyadic wavelet transform). Let the wavelet transform of the noisy observation $g(k) = x + \epsilon^{(k)}$ be denoted by

$$Y^{(k)} = X + V \tag{3}$$

The wavelet coefficients are often grouped into subbands of different scale and orientation, with one lowest frequency subband, and the rest called detail subbands. Namely, the subbands HH_j, HL_j and LH_j, $j = J_c, \dots, J-1$ correspond to the detail coefficients in diagonal, horizontal and vertical orientations, and the subband LL_{J_c} is the approximation or the smooth component. J_c is the coarsest scale of the decomposition. The main goal, in the denoising problem, is to obtain an estimate $\hat{x}$ of x from g such that the expectation of the mean-squared-error (MSE), i.e., $\mathbb{E}\left[\|x - \hat{x}\|^2 / M\right]$ is minimized. In fact, this MSE measure, which is the simple Euclidean distance between the original and denoised estimated image, is also commonly proposed in the denoising community in order to quantitatively measure the achieved performance improvement of a denoising technique leading to the well-known peak signal-to-noise ratio (PSNR) expressed in decibels as $20 \log_{10}\left[255/\sqrt{\mathrm{MSE}}\right]$. The optimal regularization scheme, in the minimum MSE sense, is closely related to the model of the statistical prior distribution of wavelet coefficients. Clearly, imprecise modeling of the statistical prior directly leads to deterioration in the resulting performance. It has been shown that the statistical behavior of the wavelet coefficients is successfully modeled by families of heavy-tailed distributions such as α-stable and BKF densities. We cite [22], in which a prior statistical model based on the α-stable densities, was proposed.

The authors used the finite mixture of Gaussians as a fast and numerically stable analytical approximation for α-stable densities in order to obtain closed-form expressions for their Bayesian denoiser. Our contribution extends their results to a more general situation, since the Bayesian denoiser has been already successfully applied to one set of observations. More precisely, we design two structures of fusion, in order to obtain the most noise-free copy possible. In the first structure, the Bayesian denoiser with the scale mixture approximation to the α-stable prior, is firstly applied to each noisy copy independently. So, we get a partial denoised image. The final recovered image is then obtained by computing the average of all denoised copies. This proposed structure can be viewed as a parallel distributed detection fusion (DDF) Radar system with multiple sensors and a center of fusion. Each sensor, based on the noisy observation, makes an individual decision about the presence or the absence of the target. The global decision is made based on the received individual decisions according to a specific fusion rule. While in the second structure, we compute the mean of the wavelet coefficients of all corrupted copies. This is done for each wavelet coefficient separately. We use the approximate α-stable model developed in [22] as a prior of the wavelet coefficients. We should note here, that the mean of an α-stable random variable (RV) remains a α-stable RV (see [26] for more details). Then we apply the nonparametric Bayesian denoiser to estimate the fused wavelet coefficients.

We explain in the following subsection how to extend the Bayesian denoiser to multiple corrupted copies by using two fusion structures.

B. Symmetric α-stable PDF approximation algorithm

For the sake of clarity, we report in this section the results found by Boubchir et al in [22].

Let Z be a α-stable RV, $Z \sim S_{\alpha_z}(\beta, \delta, \gamma)$. This distribution is completely defined by the four parameters α_z, β, γ and δ, where α_Z is the characteristic exponent, taking values $0 < \alpha_z \leq 2$, δ (with $-\infty < \delta \leq +\infty$) is the location parameter, β

(with $-1 < \beta \leq +1$) is the symmetric index and γ (with $\gamma > 0$) is the dispersion of the distribution. Further detail about α-stable can be found in [26], [27]. When $\beta = 0$, Z is symmetric α-stable $S\alpha S$ RV. Motivated by the above considerations, we model the signal component of the wavelet coefficients using the $S\alpha S$ distribution which is best defined by its characteristic function:

$$\phi(\omega) = \exp(j\delta\omega - \gamma|\omega|^{\alpha_z}) \tag{4}$$

The $S\alpha S$ model is suitable for describing signals that have highly non-Gaussian statistics and its parameters can be estimated from noisy observations. The $S\alpha S$ RV can be represented as the product of a Gaussian RV and a positive α-stable RV. Let $X \sim N(0, 2\gamma_x)$, that is, let X be distributed with Gaussian distribution ($\alpha_x = 2$). Let Y be a positive stable random variable, $Y \sim S_{\frac{\alpha_z}{2}}\left(-1, 0, \left(cos\left(\frac{\pi\alpha_z}{4}\right)^{\frac{2}{\alpha_z}}\right)\right)$ and independent from X. Then $Z = \sqrt{Y}X$.

The algorithm that fits a $S\alpha S$ to observed samples $\{z_m\}_{m=1,\cdots,M}$ follows the next steps:

- Step 1: Generate the characteristic function of the mixing PDF which is positive α-stable distributed.
- Step 2: Evaluate the positive stable PDF f_Y at N equally spaced points taking the inverse FFT of the characteristic function where N is the number of Gaussians.
- Step 3: The mixing function is the PDF of the random variable $V = \sqrt{Y}$ which is obtained by $h(v_i) = 2v_i f_Y(v_i^2)$.
- Step 4: After some substitutions, we obtain the analytical approximation for the $S\alpha S$ PDF

$$P_{\alpha,0,0,\sigma}(z) = \frac{\sum_{j=1}^{N} v_j^{-1} \exp\left(-\frac{z^2}{4\gamma v_j^2}\right) f_Y(v_j^2)}{\sqrt{4\pi\gamma}\sum_{j=1}^{N} f_Y(v_j^2)} \tag{5}$$

- Step 5: Use the EM algorithm to refine the approximation using the observed samples z_m.

C. Bayesian denoiser for one set of observations

As a first step in our approaches, we consider one corrupted copy of the image ($K = 1$ in 2). Different choices of loss function lead to different Bayesian rules and hence to different nonlinear wavelet shrinkage and wavelet thresholding rules. For example, it is well known that the L_1-loss function corresponds to the maximum a posteriori (MAP) estimator. However, except some special cases of $S\alpha S$ distributions (e.g. $\alpha = 2$), it is not easy to derive a general analytical form of the corresponding Bayesian shrinkage rule even with the scale mixture approximation. Alternatively, the L_2-based Bayes rules are used which correspond to posterior conditional means (PCM) estimates. The general expression, using the approximate prior PDF, of the PCM estimates of the wavelet coefficients is given as follows [22]:

$$\widehat{X} = Y_{\mathrm{PCM}}(Y|\theta) = \frac{\sum_j P(j)\frac{d\sigma_j^2}{\sigma_j^2+\sigma_\epsilon^2}\Phi(Y;\sigma_j^2+\sigma_\epsilon^2)}{\sum_j P(j)\Phi(Y;\sigma_j^2+\sigma_\epsilon^2)} \tag{6}$$

where θ is the hyperparameters set, $\theta = \left\{P(j), \sigma_j, \sigma_\epsilon^2\right\}$, and Φ is the normal noise PDF with variance σ_ϵ^2.

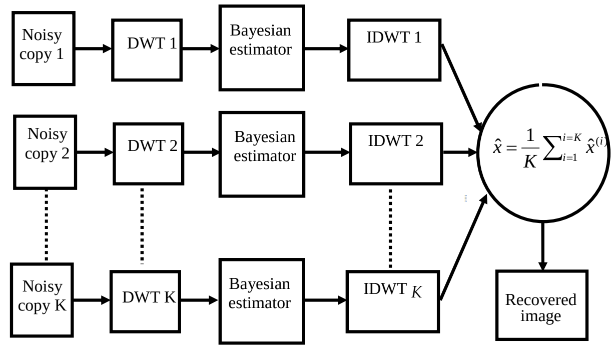


Fig. 1. First structure of fusion $\mathrm{A(B(.))}$.

D. Combining Bayesian Estimator and Averaging

1) First Structure of Fusion $\mathrm{A(B(.))}$*:* This structure for image denoising consists of three major modules: (i) a subband representation function that utilizes the wavelet transform, (ii) a Bayesian denoising algorithm based on the scale mixture approximation to the α-stable prior and (iii) the traditional averaging operation. As shown in Figure 1, the denoised copies (the out puts of the Bayesian denoiser blocs) are combined by averaging operation in order to obtain the final recovered image. This can be summarized as follows:

- First we iterate five times, the separable wavelet decomposition (as described in [1]) using Daubechies' Symmlet 8 basis wavelet. This is done for each noisy copy independently.
- Then, we model the coefficients of each subband by using the Gaussian scale mixture approximation.
- We apply the Bayesian estimator expression given in (6), to get sets of denoised wavelet coefficients.
- An inverse wavelet transform constructs the partial denoised copies $\widehat{x}^{(i)}$.
- Finally, the recovered image is obtained by computing the average of all denoised copies: $\widehat{x} = \frac{1}{K}\sum_{i=1}^{i=K}\widehat{x}^{(i)}$, where K is the number of available noisy copies.

This structure can be viewed as a parallel distributed detection fusion (DDF) Radar system with multiple sensors and a center of fusion. Each sensor, based on the noisy observation, makes an individual decision about the presence or the absence of the target. The global decision is made based on the received individual decisions according to a specific fusion rule. For the proposed structure in this paper, our fusion rule, to obtain the final noise free image, is averaging operation.

2) Second Structure of Fusion $\mathrm{B(A(.))}$*:* As shown in Figure 2, we firstly, apply the separable wavelet decomposition for each noisy copy. We then compute the average of the wavelet coefficients of all noisy copies. This is done for each coefficient independently. The Bayesian estimator expression (in (6)) is applied to the averaged wavelet coefficients in order to obtain the denoised ones. An inverse wavelet transform constructs the recovered image. We should note here that this structure requires much less computation than the first

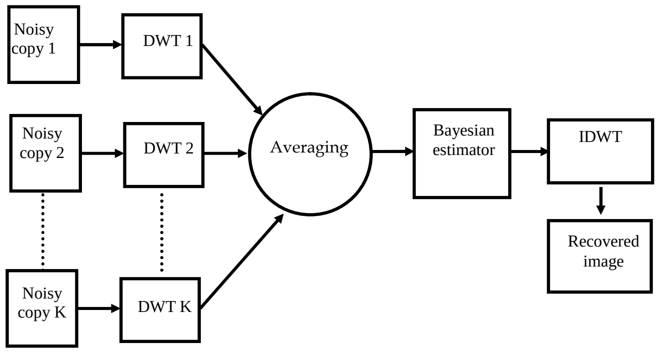
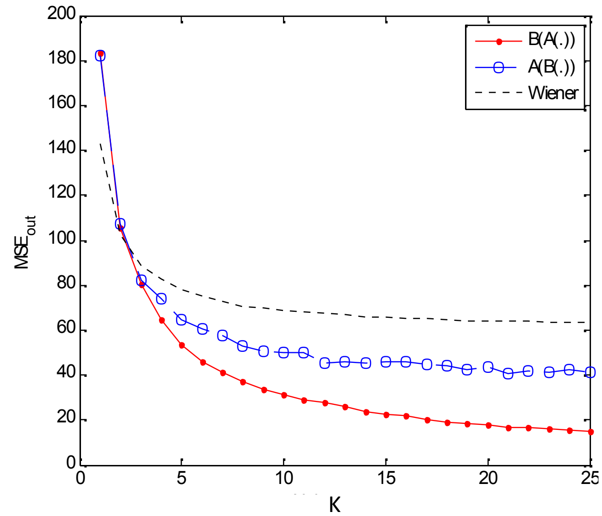
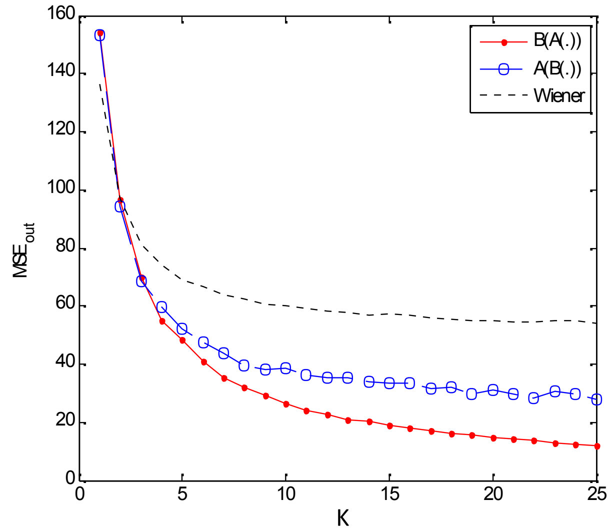


Fig. 2. Second structure of fusion $B(A(.))$.



(a) Boat image



(b) Barbara image

Fig. 3. For the test images Boat and Barbara, comparing as a function of K, the MSE of the two proposed fusion structures $(\mathrm{B}(\mathrm{A}(.))$ and $\mathrm{A}(\mathrm{B}(.)))$ and wiener filter for $\sigma_\epsilon = 20$ (an averaging of 50 simulations).

structure (since we compute the wavelet transform and the Bayesian estimation one time, whereas the latter computes it K times). However, the estimation of wavelet coefficients of each individual copy of the image may be advantageous. It is possible that a different noisy copy is collected and processed at each receiving station, and only this processed copy is kept. At a later time, these separately processed copies can be collected by a central receiver to yield one better copy.

III. Experimental results and Discussions

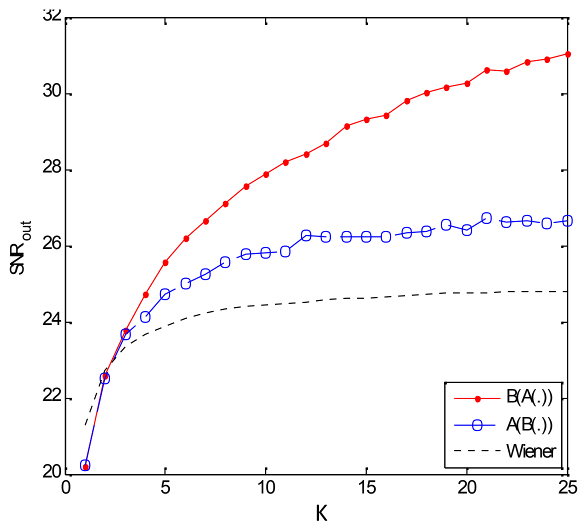
In this section, we show simulation results obtained by processing noisy copies generated from the test images: Lena, Boat, Barbara and House. In order to obtain K noisy copies, we degraded the original test images by adding the zero-mean Gaussian noise K times. We compared the results of our approach with four denoising algorithms. The first denoising algorithm is the classical Wiener filter. The Wiener filter is the wiener2 routine from the Matlab image processing toolbox, with the adaptation window size set to the default (3×3). The second denoising algorithm is the wavelet shrinkage denoising using soft thresholding [1]. The soft thresholding scheme was developed using Daubechies' Symmlet 4 mother wavelet. The maximum number of wavelet decompositions we used was five. The third denoising algorithm is the PCM Bayesian denoiser developed for one noisy copy (denoted by α-stable in the curves) [22]. The fourth denoising algorithm denoted by $\mathrm{A(T)}$ and $\mathrm{T(A)}$, was proposed in [28]. In order to quantify the achieved performance improvement, two measures were computed based on the original and the denoised data. Namely, we used the mean-square-error and the signal-to-mean-square-error commonly called the SNR. It is defined in decibels as follows:

$$\mathrm{SNR} = 10 \log_{10} \left(\frac{\sum g^2}{\sum (\hat{g} - g)^2} \right) \quad (7)$$

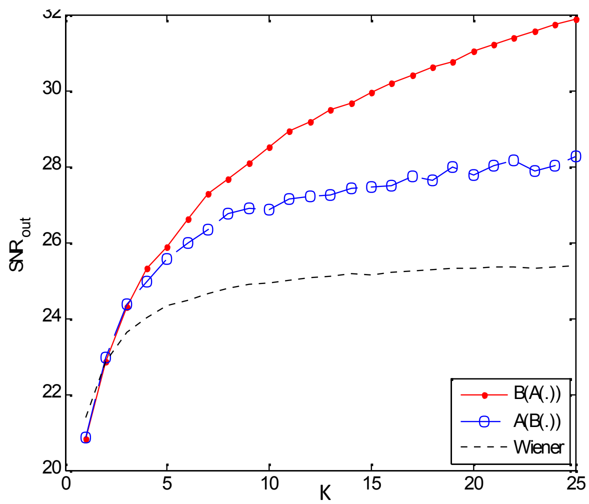
where g is the original image, and $\hat{g}$ is the recovered image. We have assumed homogeneous noise variances, i.e., the case when the noise variances are equals. The case of heterogeneous noise variances is not considered here. In Figures 3 and 4, we compare the MSE and the SNR, for K ranging from 1 to 25, of the two proposed structures of fusion and wiener filter. The test images used are Boat and Barbara.

In these figures, one can clearly see the improvement obtained by the two proposed algorithms (a lower bound for the MSE result and a higher value for the SNR), especially for $K > 5$. For $1 < K < 5$, the proposed fusion structures produce approximately the same improvements in terms of MSE and SNR. Among these denoising algorithms, the proposed structure, $\mathrm{B}(\mathrm{A}(.))$ is the best in terms of MSE and SNR, even better than $\mathrm{T}(\mathrm{A}(.))$ (see Figure 5) and better than the bayesian denoiser developed for one available noisy copy (see Figure 6).

Figure 7(a) shows the estimated images for each denoising methods for the Barbara image with $\sigma_\epsilon = 30$. One can clearly see that the visual quality of the $\mathrm{B}(\mathrm{A}(.))$ is superior to the other methods but remains comparable to the $\mathrm{A}(\mathrm{B}(.))$. This



(a) Boat image



(b) Barbara image

Fig. 4. For the test images Boat and Barbara, comparing as a function of K, the SNR of the two proposed fusion structures ($\mathrm{B}(\mathrm{A}(.))$ and $\mathrm{A}(\mathrm{B}(.))$) and wiener filter for $\sigma_\epsilon = 20$ (an averaging of 50 simulations).

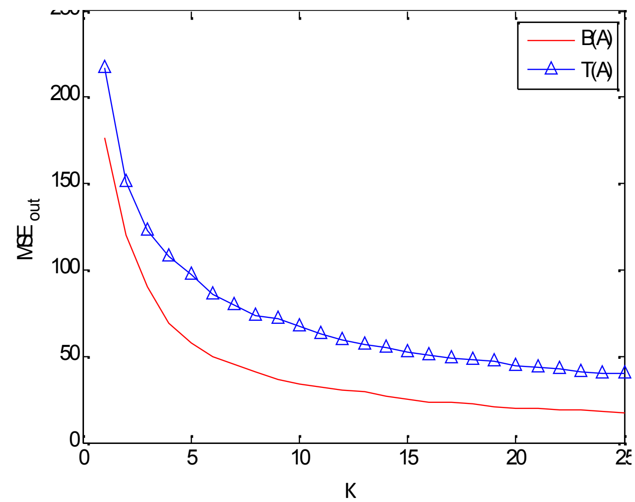


Fig. 5. MSE versus K of the denoising algorithms $\mathrm{B}(\mathrm{A}(.))$ and $\mathrm{T}(\mathrm{A}(.))$, for the test image Lena, $\sigma_\epsilon = 25$.

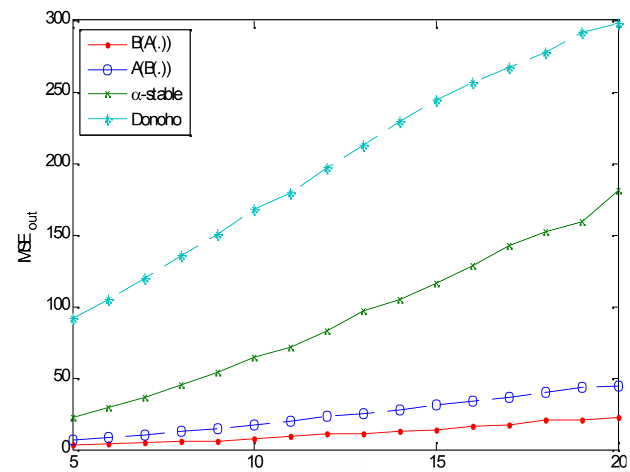


Fig. 6. MSE versus σ_ϵ, for the test image Boat with $K = 15$, of the denoising algorithms: $\mathrm{B}(\mathrm{A}(.))$, $\mathrm{A}(\mathrm{B}(.))$, Bayesian denoiser ($\alpha$-stable) and soft thresholding (Dohoho [1]).

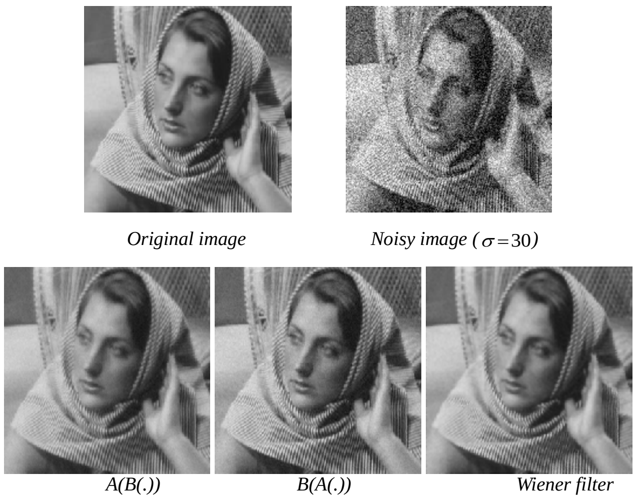
general behavior is also observed on Boat and House test images (Figure 7(b) and (c)). The $\mathrm{B}(\mathrm{A}(.))$ algorithm requires the least amount of computation since it can be implemented with only one wavelet transform and one Bayesian estimation and seems to work well for large values of σ_ϵ as well. Thus, in practice, $\mathrm{A}(\mathrm{B}(.))$ method suffices to use Bayesian estimation approaches to combine multiple noisy copies.

IV. Conclusion and Perspectives

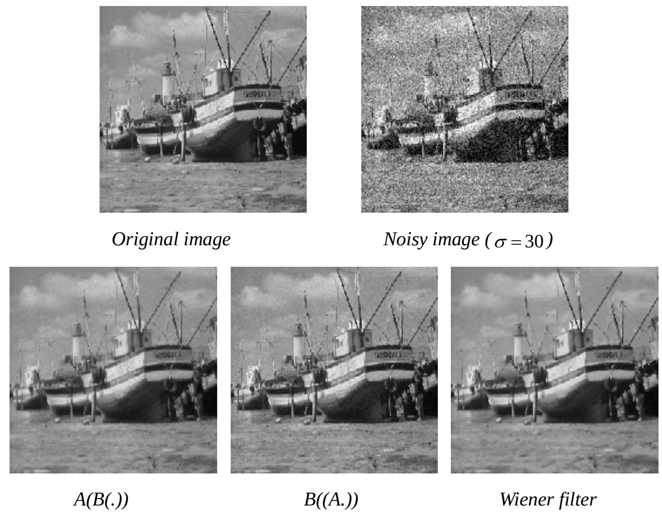
In this paper, we have presented new denoising algorithms which are based on a nonlinear nonparametric Bayesian estimator in the orthogonal wavelet domain, in the case of multiple noisy copies. We explored the idea of combining the Bayesian denoiser developed in [22] with the more traditional averaging operation by using two fusion structures. We have used the PCM Bayesian denoiser in our fusion structures because of its closed-form expression. We can note that the proposed structures perform very competitively among the wavelet-based denoising state-of-the-art methods and competitively among the Bayesian estimators developed for one noisy copy. The performances of $\mathrm{B}(\mathrm{A}(.))$ structure are superior to those of $\mathrm{A}(\mathrm{B}(.))$ structure in terms of MSE, SNR and visual quality. For computational reason, averaging followed by Bayesian estimation is recommended.

Our future work will focus on the application of other Bayesian estimators (such as the PCM and MAP Bayesian estimators based on the BKF densities [16], [17], [19]) with the oriented sparse multiscale transforms, mainly the geometrical X-lets (such as curvelets, contourlets and bandelets) in the context of proposed structures.

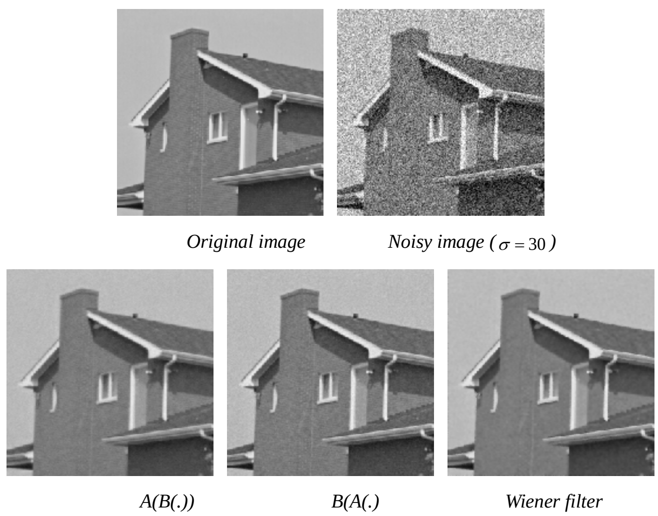
Furthermore, there is ongoing work on the application of this method in the denoising of optical soundtracks of old movies [29], [30].



(a) Barbara image



(b) Boat image



(c) Lena image

Fig. 7. Visual comparison of various denoising methods on the test images: Barbara, Boat and Lena, with $K = 25$ (over 50 runs).

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